BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION

DISTANCE LEARNING MODULE
MATHEMATICS 2

VARIATIONS

U N I T VI

On the shoreline of Bangui, Ilocos Norte, windmills were strategically placed to catch the wind that would supply electricity in the northern region of Ilocos. The power (P) generated by a windmill is directly proportional to the cube of the wind speed (w).

As you go over this unit, you will learn to analyze various relationships of quantities in real life and how decisions were made to solve these relationships. The discussions of each lesson will help you work through the solution of problems involving direct, inverse, direct square, joint and combined variations.

LESSON 1 Direct Variation

Tin cans of beverages are collected for recycling purposes in many places in the Philippines. Junk shops pay P60.00 for every kilo of tin cans bought to collectors. If c is the cost in peso and n is the number of tin cans. Then,

$$c = 60n.$$

When the number of cans is doubled, you are paid doubled. Tripling this number will also triple your earnings.

We can say that the cost c varies directly as the number of cans n and 60 in the example is a constant. The said relation is what we call a ***direct variation***.

> Direct Variation
>
> Two quantities x and y vary directly if and only if there is a constant of proportionality k such that $y = kx$ and $k \neq 0$.

The expressions,

"y varies directly as x" ,
"y is directly proportional to x" and
"y is proportional to x"

has all the same meaning. The expressions are translated in mathematical symbols as

$$y = kx,$$

where k is the constant of variation.

Variation statements can be translated into mathematical statements. The distance *d* traveled by a bicycle is proportional to the time *t* spent is translated into mathematical statement as $d = kt$, where k is the constant of variation.

Example:

The following statements are written as equations.

1. The distance (d) traveled by a car varies directly as the rate (r).

$$d = kr$$

2. The circumference (c) of a circle is proportional to the diameter (d) of a circle.

$$c = kd$$

Let's Practice for Mastery 1:

Write the following statements as an equation using k as a constant of variation:

1. The area (A) of a rectangle varies directly as the length (l) of a rectangle.
2. The weight (W) of an object varies as its mass (m).

3. The power (P) generated by a windmill is directly proportional to the cube of the wind speed (w).
4. The circumference (C) of a circle varies directly as its diameter (d).
5. The kinetic energy K exerted by a body is directly proportional to its mass (m).

Let's Check Your Understanding 1:

Write the following statements as an equation using k as a constant of variation:

1. The area (A) of a parabola is proportional to its base (b).
2. The volume (V) of a sphere varies directly as the cube of its radius (r).
3. The area (A) of a triangle varies directly as its height (h).
4. The distance (D) traveled by a moving vehicle varies directly as its rate (r).
5. The volume (V) of a right circular cylinder varies directly as the square of its radius (r).

When there is direct variation the variation constant k can be found if one pair of values of x and y is known. After that other pairs of values can also be found.

Examples:

1. If y varies directly as x and that $y = 32$ when $x = 4$. Find the constant of variation and the equation of variation.

Solution:

Express the statement "y varies directly as x", as $y = kx$. To find the constant of variation, substitute the given values in the equation.

$$32 = k(4)$$
$$k = \frac{32}{4}$$
$$k = 8$$

The variation constant is 8 and the equation of variation is given by:

$$y = 8x$$

2. Find the equation of variation when y varies directly as x and $y = 7$ when $x = 25$.

Solution:

$$y = kx$$
$$7 = k(25) \qquad \text{Substitute the given.}$$
$$k = \frac{7}{25} \text{ or}$$
$$k = 0.28$$

Therefore the equation of the variation is $y = \frac{7}{25}x$ or $y = 0.28x$.

3. If y varies directly as x and y = 6 when x = 5, what is the value of x when y = 12?

This type of problem can be solved in two ways.

Solution 1.

Find the value of k. Since y = kx, and y = 6 when x = 5, then,

$$6 = k(5)$$
$$\frac{6}{5} = k$$

Use $k = \frac{6}{5}$ to solve for x when y = 12.

$$y = kx$$
$$12 = \frac{6}{5}x$$
$$\left(\frac{5}{6}\right)12 = \left(\frac{5}{6}\right)\frac{6}{5}x$$

Multiply both sides by the multiplicative inverse of $\frac{6}{5}$.

$$10 = x$$

Solution 2.

Since $k = \frac{y}{x}$ and $k = \frac{6}{5}$, then a proportion can be made such that

$$\frac{y}{x} = \frac{6}{5}.$$

Solving for x when y = 12.

$$\frac{12}{x} = \frac{6}{5}$$

$$60 = 6x$$
$$10 = x$$

The answer is when y = 12, x = 10.

The second method illustrates a proportion, $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ which you can also use.

Let's Practice for Mastery 2.

Suppose two variables vary directly:

1. If y = 30 when x = 6, find the value of k.
2. If y = 16 when x = 24, find the value of k.
3. If y = 27 when x = 3, find the value of y when x = 2.

4. If y = 42 when x = 7, find the value of x when y = 30.
5. If y = $\frac{2}{3}$ when x = $\frac{4}{3}$, find the value of y when x = 3.

Let's Check Your Understanding 2:

Suppose two variables vary directly, find the value of k:

1. If y = 36 when x = 9, find the value of k.
2. if y = 10 when x = 15, find the value of k.
3. If y = 24 when x = 8, find the value of y when x = 6.
4. If y = 14 when x = 21, find the value of x when y = 12.
5. If y varies directly as x, and y = 6 when x = ½. What is x if y = 12?

You can use these equations to solve real life problems involving direct variations.

Example:

1. The cost c varies directly as the number n of pencils. If 5 pencils cost P23.75, what is the cost of 12 pencils?

Solution:

a. Express the statement into an equation.

$$c = kn$$

b. Solve for k:

$$k = \frac{c}{n}$$
$$= \frac{23.75}{5}$$
$$k = 4.75$$

c. Solve for the cost of 12 pencils by substituting k in c = kn.

$$c = kn$$
$$c = 4.75(12)$$
$$c = 57$$

The cost of 12 pencils is P57.00.

2. The distance (d) covered by a car varies directly as the time (t) spent traveling. If a car travels a distance of 80 kilometers in 2 hours, find the distance it will travel in 5 hours.

Solution:

a. Write the statement into an equation.

$$d = kt$$

b. Solve for the constant of variation k.

$$k = \frac{d}{t}$$
$$k = \frac{80}{2}$$
$$k = 40 \text{ km/hr}$$

c. Substituting the value of k in d = kt, solve for the distance if t = 5.

$$d = kt$$
$$d = 40(5)$$
$$d = 200 \text{ km}$$


Let's Practice for Mastery 3:

Solve the following:

1. The cost c of recycled tin cans varies directly as its weight w in kilograms. If a junk shop pays P60 per kilo of tin cans, how many kilos should a junk shop pay an amount of P390?
2. The distance d a car can travel varies directly as the rate r. If a car travels a distance of 2 km at the rate of 60 km per hour, find the distance it will travel at a rate of 40 km per hour?
3. The circumference of a circle C varies directly as the radius r of the circle. The circumference is 10π cm, when the radius if 5 cm. Write an equation and determine the constant of variation.
4. The shadow s of an object varies directly as its height h. A man 1.8 m tall casts a shadow 4.32 m long at the same time a flagpole casts a shadow 12.8 m long. How high is the flagpole?
5. A teacher charges P500 per hour of tutorial service. If she spent 3 hours tutoring per day, how much would she receive in 30 days?


Let's Check Your Understanding 3**:**

Solve the following:

1. If 5 sheets of paper cost P2.50, how much would 25 sheets cost?
2. A cyclist rode 3.5 km in 18 minutes. At that rate, how many kilometers will be traveled in 9 minutes?
3. A man deposited in his savings a fixed sum of P500 every month. In how many months would he have saved P125,000?
4. A group of businessmen sponsored a project amounting to P100,000 which is divided equally among them. If the share of each member is P2,500, how many businessmen would have been there?
5. A storeowner bought 3 kilograms of candies for P225, how much would $5\frac{1}{2}$ kilos of candies cost?

Let's Do It 3:

Answer the questions and transfer the letter associated to each question to the box below to decode the key.

What kind of boats

Y varies directly as x. Find k when x =12 and y = 45. S	Find *k* if y = 48 and x = 36 if y varies directly as x. E	Y varies directly as x. Find y if x = 3, when y = 5 if x = 2. B
Find an equation of variation if y varies directly as x and y = 15 when x = 4. S	Find y_1 in the proportion: $\frac{y_1}{4} = \frac{12}{5}$ L	Write an equation: The electric current I varies directly as the Voltage V. L
The perimeter of a square varies directly as its side. Find the constant of variation. S	If the constant of a direct variation is $\frac{15}{4}$, find x when y = 10. O	Write an equation: The distance (d) in km covered by a train varies directly as its speed (s) in kph. D
If the constant of variation 5, find y when x = 7. E	If y varies directly as x and y = 10 when x = 5, find y when x = 15. O	If y varies directly as x, find k in the table of values: x 2 4 6 8 V y 5 10 15 20

$\frac{15}{2}$	$\frac{48}{5}$	30	$2\frac{2}{3}$	d=ks	$\frac{5}{2}$	$\frac{4}{3}$	$y = \frac{15}{4}x$	$\frac{15}{4}$	35	*I = kv*	*K* = $\frac{p}{s}$

LESSON 6.2: Direct Square Variation

The number of ways in which one variable depend upon another is unlimited. The distance fallen (d) of a freely falling body varies directly as the square of the time (t) the object hits the ground.

> Another type of direct variation is the direct square variation. That is d varies directly as the square of x or $d = kx^2$.

The following examples are presented to you to gain skills in solving.

Examples:

1. If y varies directly as the square of x, and $y = 10$, when $x = 4$, find the constant. Find the equation of the variation.

Solution:

The statement "y varies directly as square of x" is written as

$$y = kx^2 .$$

Substitute the given values in the equation, you have:

$$10 = k(4)^2$$
$$10 = 16k$$
$$k = \frac{10}{16}$$
$$k = \frac{5}{8}$$

The variation constant is $\frac{5}{8}$ and the equation of variation is given by $y = \frac{5}{8}x^2$.

2. If y varies directly as the square of x and $y = 3$ when $x = 2$. Find y when $x = 7$.

Solution:

The equation suggests $y = kx^2$. Substitute the first set of values in

$$y = kx^2$$
$$3 = k(2)^2$$
$$3 = 4k$$
$$k = \frac{3}{4}$$

The equation of the variation is $y = \frac{3}{4}x^2$.

Solving for y when $x = 7$:

$$y = \frac{3}{4}(7)^2$$

$$y = \frac{3}{4}(49)$$

$$y = \frac{147}{4} \quad \text{or}$$

$$y = 36\frac{3}{4}$$

Here are some problems where you can apply direct square variations in real life situations.

1. The force F of the wind varies directly as the square of the wind velocity V. If the force exerted by the wind is .08 kg per square meter when the wind velocity is 32 km per hr., find the wind velocity when the force is doubled.

   Solution:

   Substitute the given data in the equation: $F = kv^2$

$$F = kv^2$$
$$.08 = k(32)^2$$
$$.08 = 1024k$$
$$k = \frac{.08}{1024}$$
$$k = \frac{1}{12900}$$

   When the force is doubled the wind velocity is solved as:

$$2F = kv^2$$
$$2(.08) = \frac{1}{12900}v^2$$
$$.16(12900) = v^2$$
$$2064 = v^2$$
$$\sqrt{2064} = v$$
$$v = 45.4 \text{ km per hr.}$$

Let's Practice for Mastery:

A. Write an equation to describe these situations.

1. A varies directly as the square of p.
2. T varies directly as the square of a.

3. N varies directly as s^2.
4. The area A of a circle varies directly as the square of its radius *r*.
5. The surface area A of a sphere varies directly as the square of the radius r.

B. Solve:

1. If A varies as x^2 and A = 15 when x = 5. Find A when x = 7.
2. If y varies directly as the square of x and y = 18 when x = 3. Find y when x = 2.
3. If y varies directly as the square of x and y = 625 when x = 25, find x when y = 100?
4. A varies directly as the square of b and $A = 8$, when $b = 2$. Find A when $b = 7$.
5. P varies directly as the square of q. If $P = 4$ when $q = \sqrt{3}$, what is q when $P = 48$?

Let's Check Your Understanding:

A. Write an equation to describe these situations.

1. The area A of a square varies directly as the square of it side *s*.
2. If a varies directly as the square of b and $a = 72$ when $b = 3$, find the constant of proportionality *k*.
3. If y varies directly as the square of x and y = 24 when x = 2, find y when x = 3.
4. If r varies directly as the square of s and r = 3 when s = $\sqrt{3}$, what is s when r = 36?
5. If y varies directly as the square of x and y = 4 when x = 4, find y when x = 8.

B. Solve.

2. The period t of a pendulum varies directly as the square root of the length of the pendulum. If the length of the pendulum is 3.5 m has a period of 3 seconds, what is the period of a pendulum with a length of .5m?

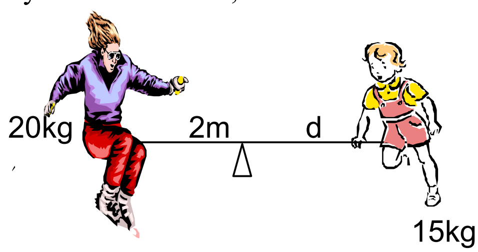
3. The surface area S of a sphere varies directly as the square of its radius r. A sphere with a diameter of 28 cm has the surface area of 784π cm^2. What is the surface area of sphere with a radius that is half as long?

4. The surface area A of a cube varies directly as the square of the length e of the edge of the cube. If the area is 37.5 cm^2 when the edge is 2.5 cm, what is the area when the edge is 3 cm?

5. A ball which falls 80 m from a building in reached the ground in 4 seconds. If the distance (d) fallen varies directly as the square of the time (t), how long will it take the ball to fall 500 m from the ground?

LESSON 6.3 INVERSE VARIATION

Jamie and Andrea are figuring out a way to balance themselves on a see-saw. Jamie who weighs 20 kilograms sits 2 meters from the center. Andrea who weighs 15 kilograms tried sitting at different distances to find the exact location she could balance Jamie. If you were Andrea, how far from the center should you sit?



To balance the weight (w) of Jamie, Andrea has to sit at a distance farther from the center.

You will notice that the relation of the weight and distance do not exhibit direct variation. In the illustration above, as the weight decreases, the distance increases and vice versa. This type of relationship is called an ***inverse variation***.

The relation shows that the distance (d) varies inversely as the weight (w). The statement is translated into a mathematical equation as $d = \frac{k}{w}$.

Two quantities vary inversely if and only if there is a constant k, $k \neq 0$, such that $y = \frac{k}{x}$ or $xy = k$.

As in direct variation, inverse variation occurs in many situations.

Example:

1. The base (b) of a triangle varies inversely as its height (h). The statement is translated in symbols as

$$b = \frac{k}{h}.$$

2. If y varies inversely as x and y = 8 when x = 9, find the value of k.

   Solution:

   a. Translate the variation statement into symbols.

$$y = \frac{k}{x}$$

   b. Solve for k:

$$y = \frac{k}{x}$$

$$9 = \frac{k}{8}$$

$$k = 72$$

Let's Practice for Mastery 1:

A. Express the following statements into a mathematical equation.

1. The length *l* of a rectangular field varies inversely as its width *w*.
2. The density *d* of air varies inversely as the volume *v* of water in the atmosphere.
3. The acceleration *a* of a car is inversely proportional to its mass *m*.
4. The velocity *v* of moving object varies inversely as its time *t*.
5. The mass *m* of a body varies inversely as the gravitational force *g*.

B. Solve:

1. If y varies inversely as x and $y = 12$ when $x = 5$, find the value of k.
2. If m varies inversely as n and $m = 18$ when $n = 3$, find the value of k.
3. If r varies inversely as s and $r = 100$ when $s = 27$, find the value of r when $s = 45$.
4. If p varies inversely as the square of q and $p = 3$ when $q = 4$, find p when $q = 16$.
5. If y varies inversely as x and $y = -2$ when $x = -8$, find x when $y = 2$.

Let's Check Your Understanding 1:

A. Express the following statements into a mathematical equation.

1. At constant temperature, the volume *V* of gas is inversely proportional to the pressure *p* exerted on it.
2. The force *F* exerted to move an object varies inversely as the distance *d*.
3. The elasticity *E* of wire varies inversely as the strain *s*.
4. The specific gravity *sp* of a substance is inversely proportional to the density *d* of water.
5. The volume *V* of a confined gas varies inversely as the pressure *p*.

B. Solve: Suppose two quantities vary inversely.

1. If y = 36 when x = 15, find the value of x when $y = 10$.
2. If y = 21 when x = 35, find the value of x when $y = 14$.
3. If m = 8 when n = 3, find m when n = 12.
4. If y = 9 when x = $\frac{5}{2}$, find y when x = $\frac{3}{5}$.
5. If a = -2 when b = -8, find a when b = 2.

Now, that you have gained analytical concept on how to deal with inverse variation, the following discussion will enhance your problem solving skills in dealing with situations that are inverse variations in nature.

Example:

The number of days required to finish a certain job varies inversely as the number of persons on the job. If 9 farmers require 10 days to plant a ricefield, how long should it take for 30 farmers to finish the same job?

Solution:

Let t - number of days required to finish the job
n - number of people on this job.

The equation: $t = \frac{k}{n}$

Solving for k, from the first set of given data,

$$10 = \frac{k}{9}$$

$$k = (9)(10)$$

$$k = 90$$

To find the time t, 30 persons should take to finish the job,

$$t = \frac{90}{n}$$

$$t = \frac{90}{30}$$

$$t = 3 \text{ days}$$

Alternative solution:

You can use $t_1 n_1 = t_2 n_2$, where t_2 is the required time for 30 persons to finish the job.

$$t_1 n_1 = t_2 n_2$$

$$(10)(9) = t_2(30)$$

$$t_2 = \frac{90}{30}$$

$$t_2 = 3$$

Let's Practice for Mastery 2:

Solve the following.

1. If 3 street sweepers can finish a certain job in 1 hr and 15 min, how long will it take 5 sweepers to finish the job?
2. A 2 kg object that is 20 cm from the fulcrum balances a 4 kg object. How far is the 4 kg object from the fulcrum?
3. The bases of triangles are inversely proportional to their altitudes. The base of a certain triangle is 14 cm and its altitude is 20 cm. Find the base of another triangle having the same area as the first, whose altitude is 28 cm.
4. The speed of a gear wheel varies inversely as the number of teeth of the gear. If a gear wheel which has 48 teeth makes 3 revolutions per minute. How many revolutions per minute will a gear wheel with 96 teeth make?
5. The frequency f or number of vibrations of a string under constant tension is inversely proportional to the length l of the string. If a 48 cm string vibrates 192 cycles per second. Find the number of vibrations per second of a 32 cm string will make.

Let's Check Your Understanding 2:

Solve the following.

1. If 5 men can finish painting a house in 10 days, how many days will it take 7 men to finish the job?
2. If 8 copying machines can complete a job order in 5 hours, how much time is required to do the same job if 2 machines are shut down?
3. Anton sits 2 m from the center of a see-saw to balance Arnold. If Arnold who weighs 30 kg sits 1.5 m from the center, what is the weight of Arnold?
4. The intensity I of light from the bulb is inversely proportional to the square of the distance s from the bulb. If I = 45 w/m^2 when the distance $s = 10$ m. Find the intensity of the light from the bulb at a distance of 5 m.
5. The organizers of a contest decided to divide equally a cash prize to 10 winners, with each receiving P50.00. If later they decided to have only 4 winners, how much would each receive?

LESSON 6.4 Joint Variation

Some physical relationships, as in area or volume, may involve three or more variables simultaneously. This lesson deals with another concept of variation, the joint variation.

> The statement a varies jointly as b and c means $a = kbc$,
>
> or $k = \dfrac{a}{bc}$, where k is the constant of variation.

Consider the area of a rectangle which is obtained from the formula $A = lw$, where the length is l and the width is w of a rectangle. The table shows the area in square centimetres for different values of the length and the base.

l	2	4	5	6	6	8	8	10
w	3	3	3	5	7	7	11	13
A	6	12	15	30	42	56	88	130

Observe that A increases as either l or w increase or both. Then it is said that the area of a rectangle varies jointly as the length and the width.

Examples:

1. Find an equation of variation where a varies jointly as b and c, and $a = 36$ when $b = 3$ and $c = 4$.

Solution:

$a = kbc$

$36 = k(3)(4)$ substitute the set of given data to find k

$k = \frac{36}{12}$ apply the properties of equality

$k = 3$

Therefore, the required equation of variation is: $a = 3bc$

2. z varies jointly as x and y. If $z = 16$ when $x = 4$ and $y = 6$, find the constant of variation and the equation of the relation.

Solution: $z = kxy$

$16 = k(4)(6)$ substitute the set of given data to find k

$k = \frac{16}{24}$ apply the properties of equality

$k = \frac{2}{3}$

The equation of the variation is: $z = \frac{2}{3}xy$.

3. The area A of a triangle varies jointly as the base b and the altitude a of the triangle. If $A = 65cm^2$ when $b = 10cm$ and $a = 13cm$, find the area of a triangle whose base is 8cm and altitude is 11cm.

Solution:

$A = kab$ the equation of the relation

$65 = k(13)(10)$ substitute the set of given data to find k

$k = \frac{65}{130}$ apply the properties of equality

$$k = \frac{1}{2}$$

The equation of the variation is: $A = \frac{1}{2}ab$

Therefore, when a = 11 and $b = 8$, the area of the triangle is

$$A = \frac{1}{2}(11)(8)$$
$$A = 44 \text{ cm}^2$$

4. The volume (V) of a prism on a square base varies jointly as the height (h) and the square of a side (s) of the base of the prism. If the volume is 81 cm^3 when a side of the base is 4*cm* and the height is 6*cm*, write the equation of the relation.

Solution:

Express the relation as:

$V = ks^2h$

$81 = k(4)^2(6)$ substitute the given values for V, *s* and *h*

$81 = k(16)(6)$

$k = \frac{81}{96}$ reduce to lowest term

$k = \frac{27}{32}$

The equation of variation is $V = \frac{27}{32}s^2h$.

5. Extending the problem on the previous example, find the volume of the prism if a side of the base is 7 cm and the height is 12 cm.

Solution:

$V = \frac{27}{32}s^2h$ from the previous example

$V = \frac{27}{32}(7)^2(12)$ substitute the given values for *s* and *h*

$V = \frac{27}{32}(49)(12)$

$V = \frac{27}{32}(588)$

$V = \frac{15876}{32}$

$V = 496.125 \text{cm}^3$

The following illustrations are applications of variation in different fields of mathematics like Geometry, Engineering, etc.

Examples:

1. The volume of a right circular cylinder varies jointly as the height and the square of the radius. The volume of a right circular cylinder, with radius 4 centimetres and height 7 centimetres, is 352 cm^3. Find the volume of another cylinder with radius 8 centimetres and height 14 centimetres.

   Solution:

   The equation of the relation is $V = khr^2$

   From the given set of data: $r = 4\ cm$, $h = 7\ cm$

   $V = 352\ cm^3$

   To find k substitute the values above:

   $V = khr^2$

   $k = \dfrac{V}{hr^2}$ rearranging the equation above

   $k = \dfrac{352}{(7)(4)^2}$

   $k = \dfrac{352}{(7)(16)}$

   $k = \dfrac{22}{7}$ simplifying the fraction

   To find the volume of a cylinder with r = 8 cm and h = 14 cm:

   $V = \dfrac{22}{7}(13)(8)^2$

   $V = \dfrac{22}{7}(14)(64)$

   $V = 2816\ cm^3$

2. The horsepower h required to propel a ship varies directly as the cube of its speed s. Find the ratio of the power required at 14 knots to that required at 7 knots.

   Solution:

   The equation of the relation is $h = ks^3$. The ratio of power required at 14 knots to 7 knots is

$$\frac{h_2}{h_1} = \frac{k(14)^3}{k(7)^3}$$

$$\frac{h_2}{h_1} = \frac{(14)^3}{(7)^3}$$ cancel out the $k's$

$$\frac{h_2}{h_1} = \frac{2744}{343}$$

$$\frac{h_2}{h_1} = \frac{8}{1}$$

3. The pressure P on the bottom of a swimming pool varies directly as the depth d of the water. If the pressure[1] is 125 Pa[2] when the water is 2 metres deep, find the pressure when it is 4.5 metres deep.

Solution 1:

$$P = kd$$

$$k = \frac{P}{d}$$ solving for the constant of variation

$$k = \frac{125}{2}$$ since P = 125 when d = 2

$$k = 62.5$$

$$P = 62.5d$$

$$P = (62.5)(4.5)$$

$$P = 281.25 \text{ Pa}$$

Solution 2:

In this solution, you do not need to find k. The equation $P = kd$ maybe written as $k = \frac{P}{d}$, meaning that the ratio $\frac{P}{d}$ is a constant.

Therefore: $$\frac{P_1}{d_1} = \frac{P_2}{d_2}$$

$$\frac{125}{2} = \frac{P_2}{4.5}$$

[1] Pressure is defined as the force exerted per unit area
[2] Pascal (Pa) is the metric unit for pressure

$$P_2 = \frac{(125)(4.5)}{2}$$

$$P_2 = 281.25 \text{ Pa}$$

Let's Practice for Mastery 1:

A. Translate each statement into mathematical statement. Use k as the constant of variation.

1. P varies jointly as q and r.
2. V varies jointly with l, w and h.
3. The area A of a parallelogram varies jointly as the base b and altitude h.
4. The volume of a cylinder V varies jointly as its height h and the square of the radius r.

B. Solve for the value of the constant of variation k, then find the missing value.

1. z varies jointly as x and y and z = 60 when x = 5 and y = 6.

   a. find z when x = 7 and y = 8
   b. find x when z = 72 and y = 4
   c. find y when z = 82 and x = 4

2. z varies jointly as x and y. If z = 3 when x = 3 and y = 15, find z when x = 6 and y = 9.
3. z varies jointly as the square root of the product x and y. If z = 3 when x = 3 and y = 12, find x when z = 6 and y = 8.
4. d varies jointly as o and g. If d = 15, when o = 14 and g = 5, find g when o = 21 and d = 8.
5. q varies jointly as r and s. If q = 2.4, when r = 0.6 and s = 0.8, find q when r = 1.6 and s = .01.

C. Solve.

1. The weight W of a cylindrical metal varies jointly as its length l and the square of its diameter d

   a. If W = 6 kg when l = 6 cm and d = 3 cm, find the equation.
   b. Find l when W = 10 kg and d = 2 cm.
   c. Find W when d = 6 cm and l = 1.4 cm.

2. The amount of gasoline used by a car varies jointly as the distance traveled and the square root of the speed. Suppose a car used 25 liters on a 100 kilometer trip at 100 km/hr. About how many liters will it use on a 192 kilometer trip at 64 km/hr?

Let's Check Your Understanding 1

A. Translate each variation statement into a mathematical statement. Use k as a constant of variation.

1. The heat H produced by an electric lamp varies jointly as the resistance R and the square of the current c.
2. The volume V of a pyramid varies jointly as the base area b and the altitude a.
3. The area A of a triangle varies jointly as one-half the base b and the altitude h.
4. The appropriate length (s) of a rectangular beam varies jointly as its width (w) and its depth (d).
5. The area A of a square varies jointly as its diagonals d_1 and d_2.

B. Solve for the constant of variation k. Solve for the missing value.

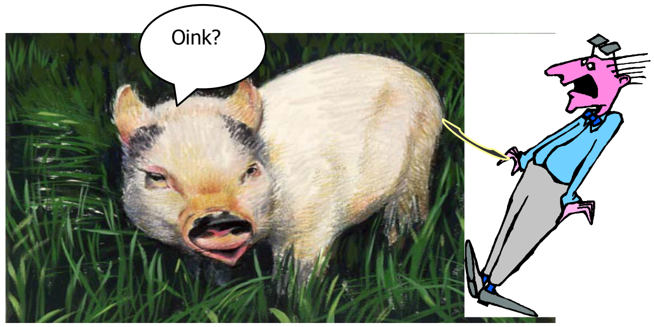
1. d varies jointly as e and l. If $d = 2.4$, when e = 0.6 and $l = 0.8$, find d when $e = 1.6$ and $l = .01$.
2. x varies jointly as w, y and z. If $x = 18$, when $w = 2$, $y = 6$ and $z = 5$, find x when $w = 5$, $y = 12$ and $z = 3$.
3. z varies jointly as x and y. $z = 60$ when $x = 3$ and $y = 4$. Find y when $z = 80$ and $x = 2$.

C. Solve.

1. The area A of rectangle varies jointly as the length l and the width w and $A = 180$ cm^2 when $l = 9$cm and w = 5cm. Find the area of a rectangle whose length is 20 cm and whose width is 5cm.
2. The weight of a rectangular block of metal varies jointly as its length, width and thickness. If the weight of a 13 by 8 by 6 dm block of aluminum is 18.7 kg, find the weight of a 16 by 10 by 4 dm block of aluminum.
3. The amount of coal used by a steamship traveling at uniform speed varies jointly as the distance traveled and the square of the speed. If a steamship uses 45 tons of coal traveling 80 km at 15 knots, how many tons will it use if it travels 102 km at 20 knots?

Let’s Do It:

What did the pig say when the man grabbed him by the tail?



Directions: Answer the questions below then transfer the letter associated to each question to the box which contains the correct answer.

I If z varies jointly as x and y, and z = 24, when x = 2 and y = 4, find z when x = 2 and y = 5	**S** If z varies jointly as x and y and z = 60 when x = 3 and y = 4. Find y when z = 80 and x = 2.
N If z varies jointly as x and y and z = 12, when x = 2 and y = 4, find the constant of variation.	**E** If w varies jointly as x and y and w = 36 when x = 3 and y = 4, find the constant of variation.
S If z varies jointly as x and y and z = 24, when x = 3 and y = 4, find z when x = 3 and y = 2.	**T** If A varies jointly as l and w and A is 36 when l = 9 and w = 2, find A if l = 6 and w = 4.
H If *a* varies jointly as *c* and *d*, and *a* = 20, when *c* = 2 and *d* = 4. Find *d* when *a* = 25 and *c* = 8.	**O** If w varies jointly as x and y^2 and w = 24 when x = 2 and y = 3, find the value of w when x = 9 and y = 4.
E If z varies jointly as x and the square of y and z = 20, when x = 4 and y = 2. Find z when x = 2 and y = 4.	**H** If A varies jointly as w^2 and l and A = 48 when w = 3 and l = 4, find the constant of variation.
T If z varies jointly as x and the square of y and z = 40, when x = 5 and y = 4. Find z when x = 4 and y = 5.	**M** z varies jointly as x and y and z = 48, when x = 4 and y = 3, find the constant of variation.
F If y varies joinly as x and z and y = 15 when x = 5 and z = 2, find the value of y if x = 14.	**I** p varies jointly as r and s and p = 32 when r = 3 and s = 2. Find the constant of variation.
E If w varies jointly as x and y and if w = 15 when x = 2 and y = 3, find the constant of variation	**D** If z varies jointly as x and y and z = 6 when x = 8 and y = 5, what is the value of z when x = 15 and y = 8?

50	$\frac{5}{4}$	30	8		$\frac{16}{3}$	12
48	$\frac{4}{3}$	3		$\frac{5}{2}$	$\frac{3}{2}$	18
	192	21		4	40	

LESSON 6.5 Combined Variation

Combined variation is another physical relationship among variables. This is the kind of variation that involves both the direct and inverse variations.

The statement t varies directly as x and inversely as y means $t = \frac{kx}{y}$, or $k = \frac{ty}{x}$, where k is the constant of variation.

This relationship among variables will be well illustrated in the following examples.

Examples:

A. The following are mathematical statements showing combined variations.

1. $k = \frac{I}{Prt}$

2. $k = \frac{E}{IR}$

3. $k = \frac{c}{ar}$

4. $\frac{Pv}{t} = k$

5. $k = \frac{ab^2}{c}$

B. Combined variation statements can be translated into a mathematical statement, using k as the constant of variation.

1. T varies directly as a and inversely as b. $\quad T = \frac{ka}{b}$

2. Y varies directly as x and inversely as the square of z. $\quad Y = \frac{kx}{z^2}$

3. P varies directly as the square of x and inversely as s. $\quad P = \frac{kx^2}{s}$

4. The time t required to travel is directly proportional to the distance d and inversely proportional to the rate r. $\quad t = \frac{kd}{r}$

5. The pressure P of a gas varies directly as its temperature t and inversely as its volume V. $P = \frac{kt}{V}$

The following examples are combined variation where some terms are unknown and can be obtained by the available information.

C. If z varies directly as x and inversely as y, and $z = 9$ when $x = 6$ and $y = 2$, find z when $x = 8$ and $y = 12$.

Solution:

The equation is $z = \frac{kx}{y}$

Substituting the given values:

$$9 = \frac{k(6)}{2}$$

$$\text{k} = \frac{9(2)}{6}$$

$$k = 3$$

Solving for z when $x = 8$ and $y = 12$:

$$z = \frac{3(8)}{12}$$

$$z = 2$$

D. x varies directly as y and inversely as z. If $x = 15$ when $y = 20$ and $z = 40$, find x when $y = 12$ and $z = 20$.

Solution:

The equation is $x = \frac{ky}{z}$

Substitute the given values to find k:

$$15 = \frac{k(20)}{40}$$

$$k = \frac{15(40)}{20}$$

$$k = 30$$

To find x when $y = 12$ and $z = 20$.

Using the equation $x = \frac{ky}{z}$

$$x = \frac{30(12)}{20}$$

$$x = 18$$

E. t varies directly as m and inversely as the square of n. If t = 16 when m = 8 and n = 2, find t when m = 13 and n = 3.

Solution:

The equation of the variation: $t = \frac{km}{n^2}$

To find k , where t = 16, m = 8 and n = 2, substitute the given values

$$16 = \frac{k(8)}{(2)^2}$$

$$k = \frac{16(2)^2}{8}$$

$$k = \frac{16(4)}{8}$$

$$k = \frac{64}{8}$$

$$k = 8$$

To find t when m = 13 and n = 3.

$$t = \frac{8(13)}{(3)^2}$$

$$t = \frac{104}{9} \quad \text{or}$$

$$t = 11\frac{5}{9}$$

F. r varies jointly as s and t and inversely as u. If $r = \frac{3}{8}$ when s = 10, t = 3 and u = 56, find r when s = 6, t = 7 and u = 84.

Solution:

The equation of the variation: $r = \frac{kst}{u}$

Substitute the given values to find k:

$$\frac{3}{28} = \frac{k(10(3)}{56}$$

$$k = \frac{3(56)}{(28)(10)(3)}$$

$$k = \frac{2}{10}$$

$$k = \frac{1}{5}$$

To find r when $s = 6$, $t = 7$ and $u = 84$.

$$r = \frac{kst}{u}$$

$$r = \frac{\frac{1}{5}(6)(7)}{84}$$

$$r = \left(\frac{42}{5}\right)\left(\frac{1}{84}\right)$$

$$r = \frac{1}{10}$$

G. Given: w varies directly as the product of x and y and inversely as the square of z. If $w = 9$ when $x = 6$, $y =27$ and $z = 3$, find w when $x = 4$, $y = 7$ and $z = 2$.

Solution:

The equation: $w = \frac{kw}{z^2}$

Substituting the first given set of values to the equation, w = 9, x = 6, y = 27 and z = 3.

$$9 = \frac{k(6)(27)}{(3)^2}$$

$$9 = \frac{k(162)}{9}$$

$$81 = 162k$$

$$k = \frac{81}{162} \quad \text{or} \quad k = \frac{1}{2}$$

Find the value of w when $k = \frac{1}{2}$ and use the second set of values when x = 4, y = 7 and z = 2, you have

$$w = \frac{kw}{z^2}$$

$$w = \frac{\frac{1}{2}(4)(7)}{2^2}$$

$$w = \frac{2(7)}{4}$$

$$w = \frac{14}{4} \quad \text{or}$$

$$w = 3.5$$

H. The current I varies directly as the electromotive force E and inversely as the resistance R. If in a system a current of 20 A flows through a resistance of 20 Ω with an electromotive force of 100 V, find the current that 150 V will send through the system.

Solution: Let I = the current in A (ampere)
E = electromotive force in V (volts)
R = Ω (ohms)

The equation: $I = \frac{kE}{R}$

Substitute the first set of given data:

$I = 20$ A
$E = 100$ V
$R = 20$ Ω

By substitution, find k:

$$20 = \frac{k100}{20}$$

$$k = \frac{(20)(20)}{100}$$

$$k = \frac{400}{100}$$

$$k = 4$$

To find how much (I) current that 150 V will send through the system

$$I = \frac{(4)(150)}{20}$$

$$I = 30$$

Notice, the system offers a resistance of 20Ω.

Let's Practice for Mastery:

A. Using k as the constant of variation, write the equation of variation for each of the following.

1. W varies jointly as the square of a and c and inversely as b.
2. The electrical resistance (R) of a wire varies directly as its length (l) and inversely as the square of its diameter (d).

3. The acceleration A of a moving object varies directly as the distance d it travels and varies inversely as the square of the time t it travels.

4. The heat H produced by an electric lamp varies jointly as the resistance R and the square of the current C.

5. The mass m of an object varies jointly as the kinetic energy E of the moving object and inversely as the square of the velocity v.

B. Solve the following

1. If r varies directly as s and inversely as the square of u, then $r = 2$ when $s = 18$ and $u = 2$. Find:

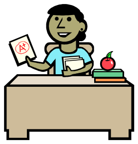
   a. r when $u = 3$ and $s = 27$.
   b. s when $u = 2$ and $r = 4$
   c. u when $r = 1$ and $s = 36$

2. p varies directly as q and the square of r and inversely as s.

   a. write the equation of the relation
   b. find k when $p = 40$, $q = 5$, $r = 4$ and $s = 6$
   c. find p when $q = 8$, $r = 6$ and $s = 9$
   d. find s when $p = 10$, $q = 5$ and $r = 2$.

3. w varies directly as xy and inversely as v^2 and $w = 1200$ when $x = 4$, $y = 9$ and $v = 6$. Find w when $x = 3$, $y = 12$ and $v = 9$.

4. Suppose p varies directly as b^2 and inversely as s^3. If $p = \frac{3}{4}$ when $b = 6$ and $s = 2$, find b when $p = 6$ and $s = 4$.



LET'S SUMMARIZE

1. Direct Variation

Two quantities vary directly if and only if there is a constant of proportionality k such that $y = kx$ and $k \neq 0$.

2. Direct Square Variation

The statement *d* varies directly as the square of *x* is translated as $d = kt^2$.

3. Inverse Variation

Two quantities vary inversely if and only if there is a constant k, $k \neq 0$, such that $y = \frac{k}{x}$ or $xy = k$.

4. Joint Variation:

The statement a varies jointly as b and c means $a = kbc$, or $k = \frac{a}{bc}$, where k is the constant of variation.

5. Combined Variation:

The statement t varies directly as x and inversely as y means $t = \frac{kx}{y}$, or $k = \frac{ty}{x}$, where k is the constant of variation.

UNIT TEST

A. The following are formulas and equations which are frequently used in mathematics and in science. State whether the relationship is considered direct, inverse, joint or combined variation.

1. C = 2π
2. A = lw
3. D = rt
4. I = prt
5. V = lwh
6. A = πr^2
7. V = πr^2
8. E = mc^2
9. F = ma
10. V = $\frac{2\pi}{T}$
11. K = $\frac{1}{2}mv^2$
12. P = $\frac{F}{A}$
13. W = Fd
14. V = IR
15. Q = mct

B. Represent each statement below by an equation:

1. The cost *c* of mangoes varies directly as its weight *w* in kilograms.
2. The area *A* of a circle is directly proportional to π and the square of its radius *r*.
3. The force F of attraction between two bodies varies inversely as the square of their distance *d*.
4. The time *t* required to finish a certain job varies inversely as the number of men *n* doing the job.
5. The volume *V* of a jewelry box varies jointly as the product of its length *l*, width *w* and height *h*.

C. For each given equation with *k* as the constant of variation. Solve for the unknown value.

1. If *y* varies directly as *x* and *y* = 12 when *x* = 4, find *y* when *x* = 12.
2. If *y* varies directly as *x* and *y* = -81 when *x* = 9, find *y* when *x* = 7.
3. If *y* varies inversely as *x*, and $y = \frac{1}{5}$ when *x* = 9, find *y* when *x* = -3.
4. If *w* varies inversely as *z* and *w* is 4 when *z* is 6. What is *z* when *w* is 14?
5. *W* varies directly as u^2 and *v*, and *W* is 75 when *u* is 5 and *v* is 9. What is *v* when *W* is 150 and *u* is 10?
6. *P* varies as the product *r* and inversely as the square of *t*. What is *t* when *P* = 2, r = 33?

7. If z varies jointly as x and y, $z = 36$ when $x = 3$ and $y = 2$, the constant of variation is
8. x varies jointly as y and z. x is 4 when y is 3 and z is 2. What is z if x is 8 and y is 10?
9. m varies directly as n but inversely as p. What is n if m is 16 and p is 18?
10. p varies inversely as q and r, and $p = \frac{2}{3}$ when q is 4 and r is 14. What is q when p is 6 and r is 10?
11. F varies directly as g and inversely as the square root of the product of I and h, and $F = 5$ when $g = 7.5$, $I = 2$ and $h = 18$. What is F when $g = 4$, $I = \frac{1}{4}$ and $h = 16$?
12. S varies directly as t and inversely as u^2, and S is 9 when t is 4 and u is 12. What is t when u is 8 and S is 16?

D. Solve the following problems.

1. The amount of income that Karen earns varies directly as the number of days she works. If she earns P8000 working in 20 days, how much would she earn if she worked 3 times as long?

2. Jorge can harvest mangoes in a farm in 8 hrs. and Adrian can do the same job in 6 hrs. How long will it take them to finish the job together?

3. When a body falls from rest, its distance from the starting point is directly proportional to the square of the time during which it has been falling. In 1.5 seconds, a body falls through 18.6 meters. How far will it fall in 5 seconds?

4. The area A of a trapezoid varies jointly as the sum of the bases b and B, and the height h. Find the constant of variation k if A =100, b =15 and B = 10 and h = 8.

5. The force of attraction, F of a body varies directly as its mass, m and inversely as the square of the distance, d from the body. When m = 8 kg and d = 5m, F = 100 Newton, find F when m = 2kg and d = 15 meters.

ANSWER KEY

Lesson 6.1

Let's Practice for Mastery 1

1. $A = kl$
2. $W = km$
3. $P = kw^2$
4. $C = kd$
5. $K = km$

Let's Check Your Understanding 1

1. $A = kb$
2. $V = kr^3$
3. $A = kh$
4. $D = kr$
5. $V = kr^2$

Let's Practice for Mastery 2

1. $k = 5$
2. $k = \frac{2}{3}$
3. $y = 18$
4. $x = 6$
5. $y = \frac{3}{2}$

Let's Check your Understanding 2

1. $k = 4$
2. $k = \frac{2}{3}$
3. $y = 18$
4. $x = 18$
5. $x = 1$

Let' Practice for Mastery 3

1. $6\frac{1}{2}$ kilos
2. $1\frac{1}{3}$ km
3. $C = kr$; $k = 2\pi$
4. $5\frac{1}{3}$ m
5. P5,000

Let's Check Your Understanding 3

1. P12.50
2. $1\frac{3}{4}$ km
3. 250 months
4. 40
5. P412.50

Let's Do It

BLOOD VESSELS

Lesson 6.2

Let's Practice for Mastery

A.

1. $A = kp^2$
2. $T = ka^2$
4. $A = kr^2$
5. $A = kr^2$

3. $N = ks^2$

B.

1. $A = \frac{147}{5}$
2. $y = 8$
3. $x = 10$
4. $A = 98$
5. $q = 6$
6. 270 minutes

Let's Check Your Understanding

A.

1. $A = ks$
2. $k = 8$
3. $y = 54$
4. $s = 6$
5. $y = 16$

B.

1. $t = \frac{3\sqrt{7}}{7}$
2. $s = 196\pi$
3. $A = 54$
4. $t = 25$

Lesson 6.3

Let's Practice for Mastery 1

A.

1. $l = \frac{k}{w}$
2. $d = \frac{k}{v}$
3. $a = \frac{k}{m}$
4. $V = \frac{k}{t}$
5. $m = \frac{k}{g}$

B.

1. $k = 60$
2. $k = 54$
3. $r = 60$
4. $p = \frac{3}{16}$
5. $x = 8$

Let's Check Your Understanding 1

A.

1. $V = \frac{k}{p}$
2. $F = \frac{k}{d}$
3. $E = \frac{k}{s}$
4. $sp = \frac{k}{d}$
5. $V = \frac{k}{p}$

B.

1. $x = 54$
2. $x = 52.5$
3. $m = 2$
4. $y = \frac{75}{2}$
5. $a = 8$

Let's Practice for Mastery 2

1. 45 min
2. 10 cm
3. 10 cm
4. 1.5
5. 288

Let's Check Your Understanding 2

1. $7\frac{1}{7}$ days
2. $6\frac{2}{3}$ days
3. 22.5 kg
4. 180 $\frac{w}{m^2}$
5. P125

Lesson 6.4

Let's Practice for Mastery 1

A.

1. P = kqr
2. V = klwh
3. A = kbh
4. $V = khr^2$

B.

1. a. Z = 112
   b. x = 270
   c. 10.25
2. $z = \frac{18}{5}$
3. x = 18
4. $g = \frac{16}{9}$
5. q = .08

C.

1. a. $w = kld^2$
   b. l = 22.5
   c. w = 56 kg
2. 38.4 liters

Let's Check Your Understanding 1

A.

1. $H = kRc^2$
2. V = kab
3. $A = \frac{kbh}{2}$
4. s = kwd
5. $A = kd_1d_2$

B.

1. k = 5; d = .08
2. $k = \frac{3}{10}$; x = 54
3. k = 5; y = 8

C.

1. 400 cm2
2. 19.18 kg
3. 136 liters

Let's Do It

THIS IS THE END OF ME

Lesson 6.5

Let's Practice for Mastery

A.

1. $w = \frac{kac}{b}$
2. $R = \frac{kl}{d^2}$
3. $A = \frac{kd}{t^2}$
4. $H = \frac{kR}{C^2}$
5. $m = \frac{kE}{V^2}$

B.

1. a. $r = \frac{4}{3}$
   b. $s = 36$
   c. $u = 4$

2. a. $p = \frac{kqr^2}{w}$
   b. $k = 3$
   c. $p = 96$
   d. $s = 6$

3. $w = \frac{1600}{3}$
4. $b = 48$

UNIT TEST:

A.

1. direct
2. joint
3. joint
4. joint
5. joint
6. direct
7. direct
8. joint
9. joint
10. inverse
11. joint
12. combined
13. joint
14. joint
15. joint

B.

1. $C = kw$
2. $A = k\pi r^2$
3. $F = \frac{k}{d^2}$
4. $t = \frac{k}{m}$
5. $v = klwh$

C.

1. $y = 36$
2. $y = -63$
7. $k = 6$
8. $z = \frac{6}{5}$

3. $y = -\frac{3}{5}$
4. $z = \frac{12}{7}$
5. $v = \frac{9}{2}$
6. $t = \frac{\sqrt{66}}{2}$
9. $n = 288$
10. $q = \frac{28}{45}$
11. $F = 8$
12. $t = \frac{256}{81}$

D.

1. P24,000
2. $3\frac{6}{7}$ days
3. 41.3 m
4. $k = \frac{1}{2}$
5. F = 2.78 newtons