Module 2
Circles

What this module is about

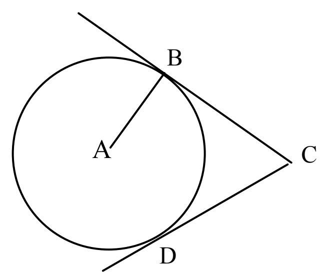
This module will discuss in detail the characteristics of tangent and secants; the relationship between tangent and radius of the circle; and how secant and tangent in a circle create other properties particularly on angles that they form. This module will also show how the measures of the angles formed by tangents and secants can be determined and other aspects on how to compute for the measures of the angles.

What you are expected to learn

This module is written for you to

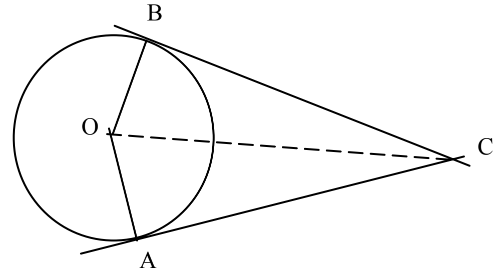
1. Define and illustrate tangents and secants.
2. Show the relationship between a tangent and a radius of a circle.
3. Identify the angles formed by tangents and secants.
4. Determine the measures of angles formed by tangents and secants.

How much do you know



If $\overline{CB}$ and $\overline{CD}$ are tangents to circle A, then

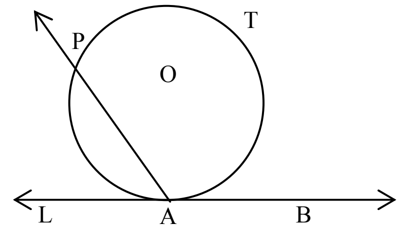
1. $\overline{CB}$ ___ $\overline{CD}$
2. $\overline{CB}$ ____ $\overline{AB}$



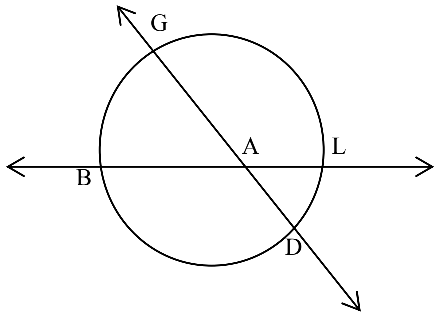
3. $\overline{CB}$ and $\overline{CA}$ are tangents to circle O.
   If $m\angle BOA = 160$, then $m\angle C =$_____.

4. If $m\angle BCO = 22$, what is $m\angle ACO$?

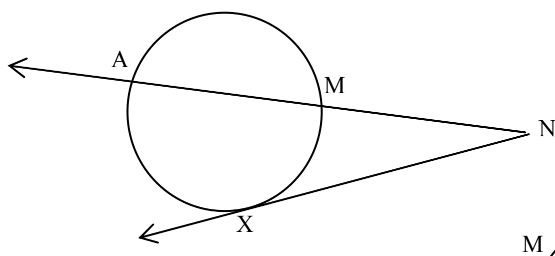
5. In the figure, if m $\overset{\frown}{PTA} = 242$, what is $m\angle PAL$?



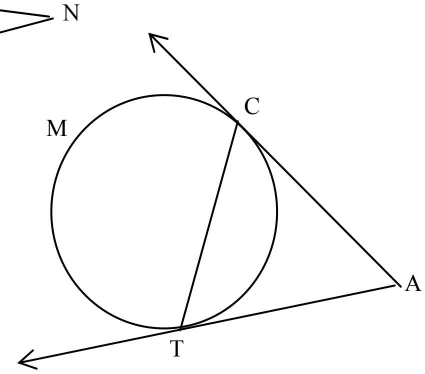
6. Two secants $\overleftrightarrow{GD}$ and $\overleftrightarrow{BL}$ intersect at A.
If $m\,\overset{\frown}{BG} = 83$ and $m\,\overset{\frown}{LD} = 39$, find $m\angle GAB$.



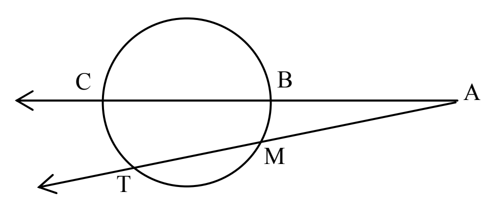
7. In the figure, if $m\,\overset{\frown}{MX} = 54$, and $m\overset{\frown}{AX} = 120$, what is $m\angle N$?



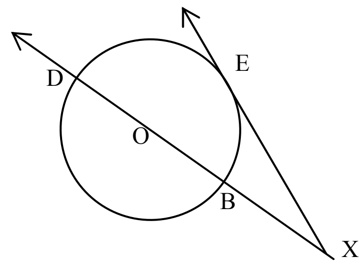
8. $\overrightarrow{AC}$ and $\overrightarrow{AT}$ are tangents to the circle with C and T as the points of tangency. If ΔACT is an equilateral triangle, find $m\,\overset{\frown}{CT}$.



9. $\overrightarrow{AC}$ and $\overrightarrow{AT}$ are secants. If $m\angle A = 23$ and $m\,\overset{\frown}{CT} = 66$, find $m\,\overset{\frown}{BM}$.



10. $\overline{DB}$ is a diameter of circle O. If the ratio of DE:EB is 3:2, what is $m\angle X$?



What you will do

Lesson 1

Circles, Tangents, Secants and Angles They Form

A line on the same plane with a circle may or may not intersect a circle. If ever a line intersects a circle, it could be at one point or at two points. The figures at the right showed these three instances.

a)

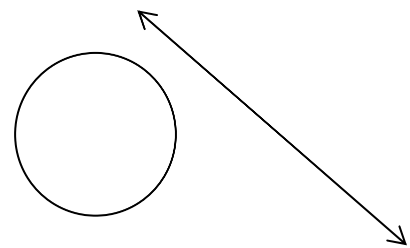


Figure a showed a line that does not intersect the circle.

b)

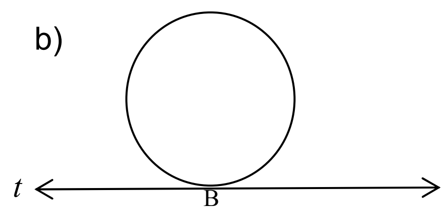


Figure b showed that line *t* intersects the circle at only one point.

c)

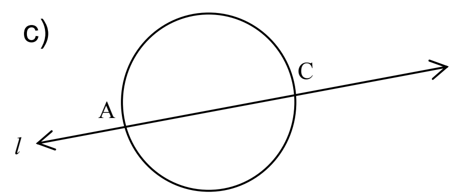


Figure c showed line *l* intersecting the circle at two points A and C.

We will focus our study on figures b and c.

In figure b, line *t* is called a <u>tangent</u> and point B is called the <u>point of tangency.</u> Therefore a ***tangent*** is a line that intersects a circle at only one point and the point of intersection is called the point of tangency.

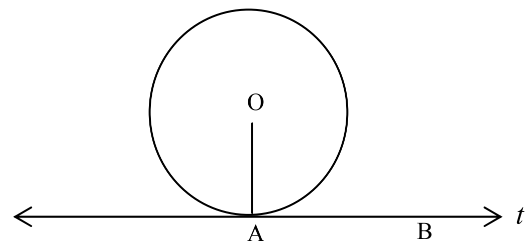
In figure c, line *l* intersects the circle at two points A and C. Hence, line *l* is called a <u>secant.</u> Thus a ***secant*** is a line that intersects a circle at two points.

Some properties exist between tangent and circle and they will be discussed here in detail. The first theorem is given below.

Theorem: Radius-Tangent Theorem. If a line is tangent to a circle, then it is perpendicular to the radius at the point of tangency.

Given: line t is tangent to circle O at A.
$\overline{OA}$ is a radius of the circle.

Prove: $t \perp \overline{OA}$



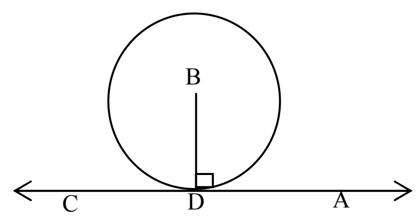
Proof:

Statements	Reasons
1. Let B be another point on line t.	1. The Line Postulate
2. B is on the exterior of circle O	2. Definition of a tangent line (A tangent can intersect a circle at only one point .
3. $\overline{OA}$ < $\overline{OB}$	3. The radius is the shortest segment from the center to the circle and B is on the exterior of the circle.
4. $\overline{OA} \perp t$	4. The shortest distance from a point to a line is the perpendicular segment.

Example:

In the figure, if $\overleftrightarrow{AC}$ is tangent to circle B, then

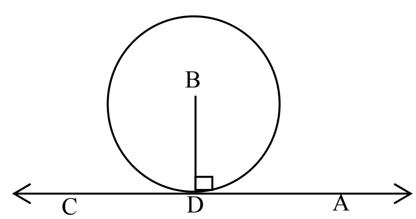
$\overleftrightarrow{AC} \perp \overline{BD}$ at D.



The converse of the theorem is also true.
Converse: The line drawn perpendicular to the radius of a circle at its end on the circle is tangent to the circle.

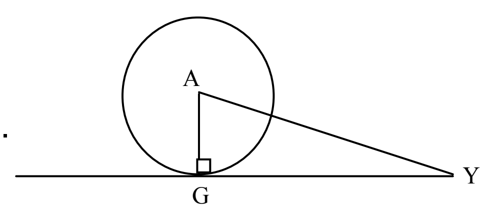
Illustration:

If $\overleftrightarrow{AC} \perp \overline{BD}$ at D, then
$\overleftrightarrow{AC}$ is tangent to circle B.



Examples:

$\overline{GY}$ is tangent to circle A.
1. What kind of triangle is Δ AGY? Give reason.
2. If $m\angle A = 79$, what is $m\angle Y$?



Solutions:

1. ΔAGY is a right triangle because $\overline{GY}$ is tangent to circle A and tangent line is perpendicular to the radius of the circle. Perpendicular lines make right angles between them thus $\angle AGY$ is a right angle making ΔAGY a right triangle.

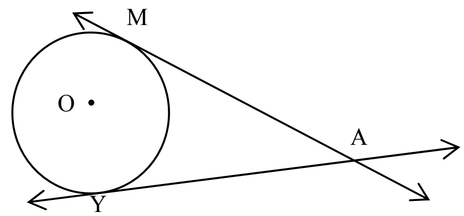
2. Since ΔAGY is a right triangle, then

$$m\angle A + m\angle Y = 90$$
$$79 + m\angle Y = 90$$
$$m\angle Y = 90 - 79$$
$$= 11$$

A circle is composed of infinite number of points, thus it can also have an infinite number of tangents. Tangents of the same circle can intersect each other only outside the circle.

At this point, we will discuss the relationship of tangents that intersect the same circle. As such, those tangents may or may not intersect each other. Our focus here are those tangents that intersect each other outside the circle.

Consider the given figure:

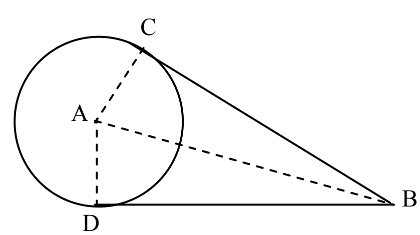


$\overline{AM}$ and $\overline{AY}$ are tangent segments from a common external point A. What relationship exists between $\overline{AM}$ and $\overline{AY}$? The next theorem will tell us about this relationship and other properties related to tangent segments from a common external point.

Theorem: If two tangent segments are drawn to a circle from an external point then

a. the two tangent segments are congruent and
b. the angle between the segments and the line joining the external point and the center of the circle are congruent.

Given: Circle A. $\overline{BC}$ and $\overline{BD}$ are two tangent segments from a common external point B. C and D are the points of tangency.



Prove: a. $\overline{BC} \cong \overline{BD}$
b. $\angle CBA \cong \angle DBA$

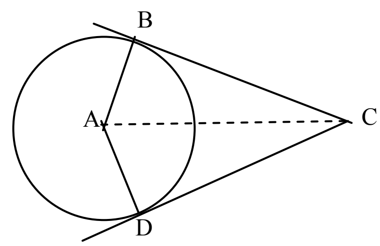
Proof:

Statements	Reasons
1. Draw $\overline{AC}$, $\overline{AD}$, $\overline{AB}$	1. Line determination Postulate
2. $\overline{BC}$ and $\overline{BD}$ are two tangent segments from a common external point B.	2. Given
3. $\overline{AC} \perp \overline{BC}$, $\overline{AD} \perp \overline{BD}$	3. A line tangent to a circle is perpendicular to the radius at the point of tangency
4. $\angle ACB$ and $\angle ADB$ are right angles	4. Definition of right angles

5. ΔACB and ΔADB are right triangles 6. $\overline{AC} \cong \overline{AD}$ 7. $\overline{BC} \cong \overline{BD}$ 8. ΔACB $\cong \Delta$ADB 9. $\overline{BC} \cong \overline{BD}$ 10. $\angle CBA \cong \angle DBA$	5. Definition of right triangles 6. Radii of the same circle are congruent 7. Reflexive property of Congruency 8. Hy L Congruency Postulate 9. Corresponding parts of congruent triangles 10. are congruent.

Examples:

a) In the figure, $\overline{CB}$ and $\overline{CD}$ are tangents to circle A at B and D.



1. If CB = 10 what is CD?
2. If $m\angle BAC = 49$, what is $m\angle BCA$?
3. $m\angle BCD = 73$, what is $m\angle BCA$? $m\angle DCA$

Solution:

1. Since $\overline{CB}$ and $\overline{CD}$ are tangents to the same circle from the same external point, then $\overline{CB} \cong \overline{CD}$, and therefore, CB = CD. Thus if CB = 10 then CD = 10

2. $m\angle BAC + m\angle BCA = 90$

$49 + m\angle BCA = 90$

$m\angle BCA = 90 - 49$

$= 41$

3. $m\angle BCA = \frac{1}{2}(m\angle BCD)$

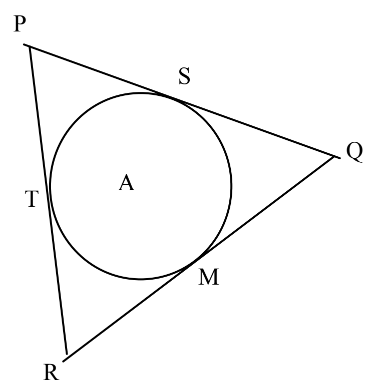
$= \frac{1}{2}(73)$

$= 36.5$

$\angle BCA \cong \angle DCA$

$m\angle BCA = m\angle DCA = 36.5$

b) $\overline{PQ}$, $\overline{QR}$ and $\overline{PR}$ are tangents to circle A at S, M and T respectively. If PS = 7, QM = 9 and RT = 5, what is the perimeter of ΔPQR?



Solution:

Using the figure and the given information, It is therefore clear that PS = PT, QS = QM and RM = RT.

PQ = PS + SQ

QR = QM + MR

PR = PT + RT

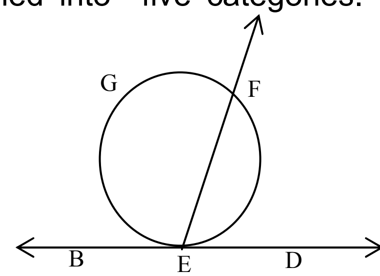
Perimeter of $\Delta PQR = PQ + QR + PR$

$$
\begin{aligned}
&= (PS + SQ) + (QM + MR) + (PT + RT)\\
&= (PS + QM) + (QM + RT) + (PS + RT)\\
&= 2PS + 2QM + 2RT\\
&= 2(PS + QM + RT)\\
&= 2(\ 7 + 9 + 5)\\
&= 2\ (21)\\
&= 42
\end{aligned}
$$

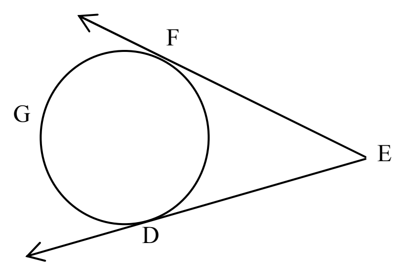
Every time tangents and secants of circles are being studies, they always come with the study of angles formed between them. Coupled with recognizing the angles formed is the knowledge of how to get their measures. The next section will be devoted to studying angles formed by secants and tangents and how we can get their measures.

Angles formed by secants and tangents are classified into five categories. Each category is provided with illustration.

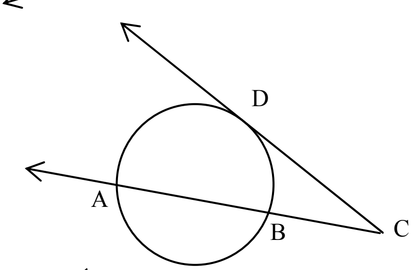
1. Angle formed by secant and tangent intersecting on the circle. In the figure, two angles of this type are formed, $\angle FED$ and $\angle FEB$. Each of these angles intercepts an arc. $\angle FED$ intercepts $\overset{\frown}{EF}$ and $\angle FEB$ intercepts $\overset{\frown}{EGF}$.



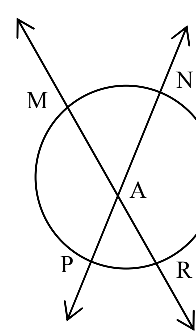
2. Angle formed by two tangents. In the figure, $\angle E$ is formed by two tangents. The angle intercepts the whole circle divided into 2 arcs, minor arc $\overset{\frown}{FD}$, and major arc $\overset{\frown}{FGD}$.



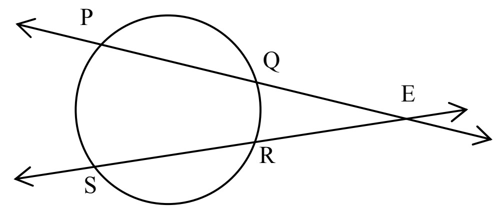
3. Angle formed by a secant and a tangent that intersect at the exterior of the circle. $\angle C$ is an angle formed by a secant and a tangent that intersect outside the circle. $\angle C$ intercepts two arcs, $\overset{\frown}{DB}$ and $\overset{\frown}{AD}$.



4. Angle formed by two secants that intersect in the interior of the circle. The figure shows four angles formed. $\angle MAN$, $\angle NAR$, $\angle PAR$ and $\angle PAM$. Each of these angle intercepts an arc. $\angle MAN$, intercepts $\overset{\frown}{MN}$, $\angle NAR$, intercepts $\overset{\frown}{NR}$, $\angle PAR$ intercepts $\overset{\frown}{PR}$ and $\angle MAP$ intercepts $\overset{\frown}{MP}$.



5. Angle formed by two secants intersecting outside the circle. $\angle E$ is an angle formed by two secants intersecting outside the circle. $\angle E$ intercepts two arcs namely $\overset{\frown}{QR}$ and $\overset{\frown}{PR}$

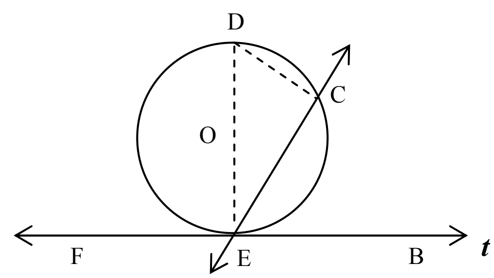


How do we get the measures of angles illustrated in the previous page? To understand the answers to this question, we will work on each theorem proving how to get the measures of each type of angle. It is therefore understood that the previous theorem can be used in the proof of the preceding theorem.

Theorem: The measure of an angle formed by a secant and a tangent that intersect on the circle is one-half its intercepted arc.

Given: Circle O. Secant *m* and tangent *t* intersect at E on circle O.

Prove: $m\angle CEB = \frac{1}{2}\overset{\frown}{CE}$



Proof:

Statements	Reasons
1. Draw diameter $\overline{ED}$. Join DC.	1. Line determination Postulate
2. $\overline{DE} \perp t$	2. Radius-tangent theorem
3. $\angle DCE$ is a right angle	3. Angle inscribed in a semicircle is a right angle.
4. $\angle DEB$ is a right angle	4. Perpendicular lines form right angles
5. ΔDCE is a right triangle	5. Definition of right triangle
6. $m\angle 1$ + $m\angle 2 = 90$	6. Acute angles of a right triangle are complementary
7. $m\angle 1 + m\angle BEC = m\angle DEB$	7. Angle addition Postulate
8. $m\angle 1 + m\angle BEC = 90$	8. Definition of complementary angles
9. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle BEC$	9. Transitive Property of Equality
10. $m\angle 1 = m\angle 1$	10. Reflexive Property of Equality
11. $m\angle 2 = m\angle BEC$	11. Subtraction Property of Equality
12. $m\angle 2 = \frac{1}{2} m\overset{\frown}{CE}$	12. Inscribed angle Theorem
13. $m\angle BEC = \frac{1}{2} m\overset{\frown}{CE}$	13. Substitution

Illustration:

In the given figure, if $m\overset{\frown}{CE} = 104$, what is the $m\angle BEC$? What is $m\angle CEF$?

Solution:

$$m\angle BEC = \tfrac{1}{2}\, m\overset{\frown}{CE}$$
$$m\angle BEC = \tfrac{1}{2}(104)$$
$$= 52$$

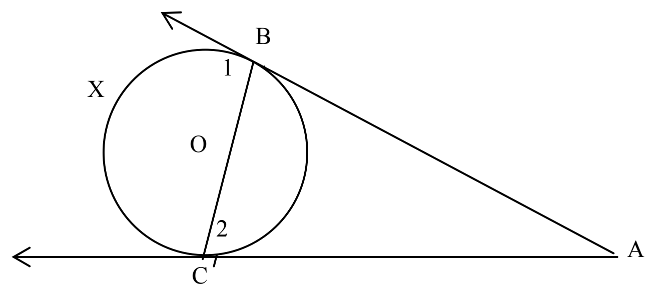
$$m\angle CEF = ½\,(m\overset{\frown}{CDE})$$
$$m\angle CEF = ½\,(360 - 104)$$
$$= ½\,(256)$$
$$= 128$$

Let's go to the next theorem.

Theorem: The measure of an angle formed by two tangents from a common external point is equal to one-half the difference of the major arc minus the minor arc.

Given: Circle O. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents

Prove: $m\angle A = \frac{1}{2}(\overset{\frown}{BXC} - \overset{\frown}{BC})$



Proof:

Statements	Reasons
1. Draw chord $\overline{BC}$	1. Line determination Postulate
2. In ΔABC, $\angle 1$ is an exterior angle	2. Definition of exterior angle
3. $m\angle 1 = m\angle 2 + m\angle A$	3. Exterior angle theorem
4. $m\angle A = m\angle 1 - m\angle 2$	4. Subtraction Property of Equality
5. $m\angle 1 = ½\, m\overset{\frown}{BXC}$ $m\angle 2 = ½\, m\overset{\frown}{BC}$	5. Measure of angle formed by secant and tangent intersecting on the circle is one-half the intercepted arc.
6. $m\angle A = ½\, m\overset{\frown}{BXC} - ½\, m\overset{\frown}{BC}$	6. Substitution
7. $m\angle A = ½(m\overset{\frown}{BXC} - m\overset{\frown}{BC})$	7. Algebraic solution (Common monomial Factor)

Illustration:

Find the $m\angle A$ if $m\overset{\frown}{BC} = 162$.

Solution:

Since $m\angle A = ½(m\overset{\frown}{BXC} - m\overset{\frown}{BC})$ then we have to find first the measure of major arc BXC. To find it, use the whole circle which is 360^{o}.

$$m\widehat{BXC} = 360 - m\widehat{BC}$$
$$= 360 - 162$$
$$= 198$$

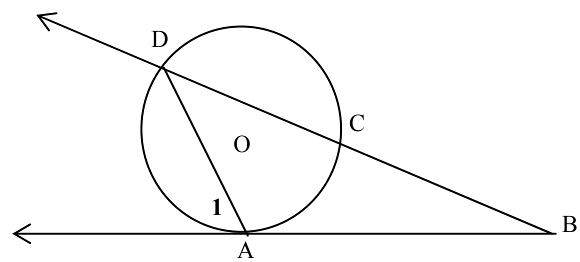
Then we use the theorem to find the measure of $\angle A$

$$m\angle A = \tfrac{1}{2}(m\widehat{BXC} - m\,\widehat{BC})$$
$$= \tfrac{1}{2}(198 - 162)$$
$$= \tfrac{1}{2}(36)$$
$$m\angle A = 18$$

We are now into the third type of angle. Angle formed by secant and tangent intersecting on the exterior of the circle.

Theorem: The measure of an angle formed by a secant and tangent intersecting on the exterior of the circle is equal to one-half the difference of their intercepted arcs.

Given: $\overrightarrow{BA}$ is a tangent of circle O
$\overrightarrow{BD}$ is a secant of circle O
$\overrightarrow{BA}$ and $\overrightarrow{BD}$ intersect at B



Prove: $m\angle B = \frac{1}{2}(\widehat{AD} - \widehat{AC})$

Proof:

Statements	Reasons
1. $\overrightarrow{BA}$ is a tangent of circle O, $\overrightarrow{BD}$ is a secant of circle O	1. Given
2. Draw $\overline{AD}$	2. Line determination Postulate
3. $\angle 1$ is an exterior angle of Δ DAB	3. Definition of exterior angle
4. $m\angle 1 = m\angle B + m\angle ADB$	4. Exterior angle Theorem
5. $m\angle B = m\angle 1 - m\angle ADB$	5. Subtraction Property of Equality
6. $m\angle 1 = \tfrac{1}{2} m\,\widehat{AD}$	6. The measure of an angle formed by secant and tangent intersecting on the circle equals one-half its intercepted arc.
7. $m\angle ADB = \tfrac{1}{2}\, m\widehat{AC}$	7. Inscribed angle Theorem
8. $m\angle B = \tfrac{1}{2}\, m\widehat{AD} - \tfrac{1}{2}\, m\widehat{AC}$	8. Substitution
9. $m\angle B = \tfrac{1}{2}\,(m\widehat{AD} - m\widehat{AC})$	9. Simplifying expression

Illustration:

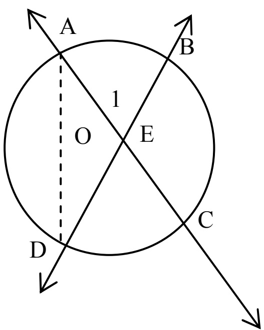
In the figure if $m\widehat{AD} = 150$, and $m\widehat{AC} = 73$, what is the measure of $\angle B$?

Solution:

$$m\angle B = \tfrac{1}{2}(m\widehat{AD} - m\widehat{AC})$$
$$= \tfrac{1}{2}(150 - 73)$$
$$= \tfrac{1}{2}(77)$$
$$m\angle B = 38.5$$

The next theorem will tell us how angles whose vertex is in the interior of a circle can be derived. Furthermore, this will employ the previous knowledge of vertical angles whether on a circle or just on a plane.

Theorem: The measure of an angle formed by secants intersecting inside the circle equals one-half the sum of the measures of the arc intercepted by the angle and its vertical angle pair.



Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are secants intersecting inside circle O forming $\angle 1$ with vertical angle pair $\angle CED$. (We will just work on one pair of vertical angles.)

Prove: $m\angle 1 (m\angle AEB) = \tfrac{1}{2}(\widehat{AB} + \widehat{DC})$

Proof:

Statements	Reasons
1. $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are secants intersecting inside circle O.	1. Given
2. Draw $\overline{AD}$	2. Line determination Postulate
3. $\angle 1$ is an exterior angle of ΔAED	3. Definition of exterior angle
4. $m\angle 1 = m\angle DAC + m\angle ADE$	4. Exterior angle Theorem
5. $m\angle DAC = \tfrac{1}{2} m\widehat{DC}$ $m\angle ADE = \tfrac{1}{2} m\widehat{AB}$	5. Inscribed Angle Theorem
6. $m\angle 1 = \tfrac{1}{2} m\widehat{DC} + \tfrac{1}{2} m\widehat{AB}$ $m\angle 1 = \tfrac{1}{2}(m\widehat{DC} + m\widehat{AB})$	6. Substitution

Illustration:

Using the figure, find the measure of $\angle 1$ if $m\widehat{AB} = 73$ and $m\widehat{CD} = 90$.

Solution:

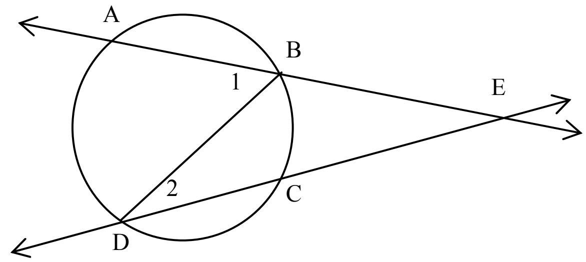
Using the formula in the theorem,

$$m\angle 1 = \tfrac{1}{2}(m\widehat{DC} + m\widehat{AB})$$
$$= \tfrac{1}{2}(90 + 73)$$
$$= \tfrac{1}{2}(163)$$
$$= 81.5$$

Let us discuss how to find the measure of the angle formed by two secants intersecting outside the circle.

Theorem: The measure of the angle formed by two secants intersecting outside the circle is equal to one-half the difference of the two intercepted arcs.

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two secants intersecting outside of circle O forming $\angle BEC$ outside the circle.



Prove: $m\angle BEC = \frac{1}{2}(\overset{\frown}{AD} - \overset{\frown}{BC})$

Proof:

Statements	Reasons
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are secants of circle O forming $\angle BEC$ outside the circle.	1. Given
2. Draw $\overline{DB}$	2. Line determination Postulate
3. $\angle 1$ is an exterior angle of Δ DBE	3. Definition of exterior angle of a triangle
4. $m\angle 1 = m\angle 2 + m\angle BEC$	4. Exterior angle Theorem
5. $m\angle BEC = m\angle 1 - m\angle 2$	5. Subtraction Property of Equality
6. $m\angle 1 = \frac{1}{2} m\overset{\frown}{AD}$ $m\angle 2 = \frac{1}{2} m\overset{\frown}{BC}$	6. Inscribed Angle Theorem
7. $m\angle BEC = \frac{1}{2} m\overset{\frown}{AD} - \frac{1}{2} m\overset{\frown}{BC}$ $m\angle BEC = \frac{1}{2}(m\overset{\frown}{AD} - m\overset{\frown}{BC})$	7. Substitution

Illustration:

Find the measure of $\angle E$ if $m\overset{\frown}{AD} = 150$ and $m\overset{\frown}{BC} = 80$.

Solution:

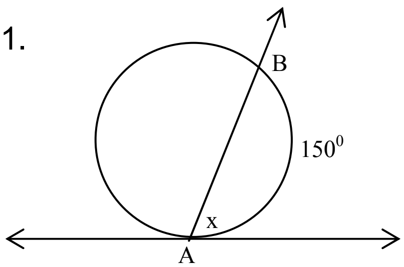
Again we apply the theorem using the formula:

$$
\begin{aligned}
m\angle BEC &= \tfrac{1}{2}(m\overset{\frown}{AD} - m\overset{\frown}{BC}) \\
&= \tfrac{1}{2}(150 - 80 \\
&= \tfrac{1}{2}(70) \\
&= 35
\end{aligned}
$$

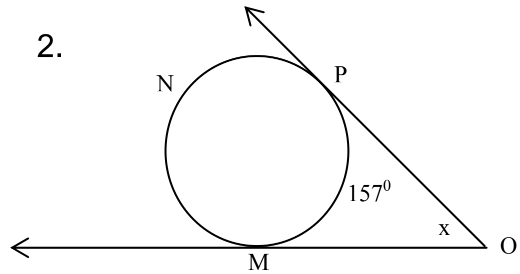
Example 1.

In each of the given figure, find the measure of the unknown angle (x).

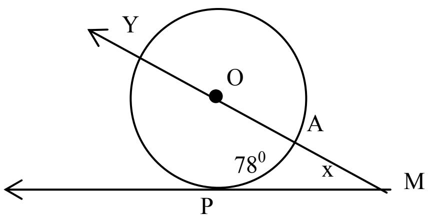
1.



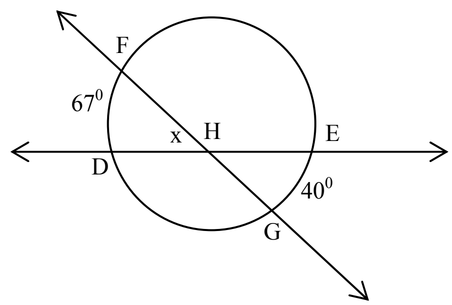
2.



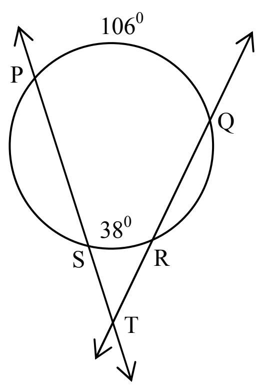
3.



4.



5.



Solutions:

1. Given: $\widehat{AB} = 150^0$
   Find: $m\angle x$

$\angle x$ is an angle formed by a secant and a tangent whose vertex is on the circle. $\angle x$ intercepts $\widehat{AB}$.

$m\angle x = ½ \widehat{AB}$
$m\angle x = ½ (150)$
$m\angle x = 75$

2. Given: $m\,\widehat{MP} = 157$

   Find: $m\angle x$

$\angle x$ is an angle formed by two tangents from a common external point. $\angle x$ intercepts minor arc MP and major arc MNP

$m\angle x = \frac{1}{2}(\widehat{MNP} - \widehat{MP})$

$m\,\widehat{MNP} + m\widehat{MP} = 360$

$m\widehat{MNP} = 360 - m\,\widehat{MP}$

$= 360 - 157$

$m\widehat{MNP} = 203$

$m\angle x = \frac{1}{2}(203 - 157)$

$= \frac{1}{2}(46)$

$= 23$

3. Given: $m\,\widehat{AP} = 78$

   $\overline{AY}$ is a diameter

   Find: $m\angle x$

Since $\overline{AY}$ is a diameter, then $\widehat{AY}$ is a semicircle and $m\,\widehat{AY} = 180$. Therefore

a. $m\,\widehat{AP} + m\,\widehat{PY} = 180$

$m\,\widehat{PY} = 180 - m\,\widehat{AP}$

$m\,\widehat{PY} = 180 - 78$

$m\,\widehat{PY} = 102$

b. $m\angle x = \frac{1}{2}(m\widehat{PY} - m\widehat{AP})$

$= \frac{1}{2}(102 - 78)$

$= \frac{1}{2}(24)$

$= 12$

4. Given: $m\widehat{FD} = 67$, $m\,\widehat{GE} = 40$

   Find: $m\angle x$

$\angle x$ is an angle formed by secants that intersect inside the circle, Hence

$m\angle x = \frac{1}{2}(m\widehat{FD} + m\widehat{GE})$

$= \frac{1}{2}(67 + 40)$

$= \frac{1}{2}(107)$

$= 53.5$

5. Given: $m\widehat{SR} = 38$, $m\widehat{PQ} = 106$

Find: $m\angle x$

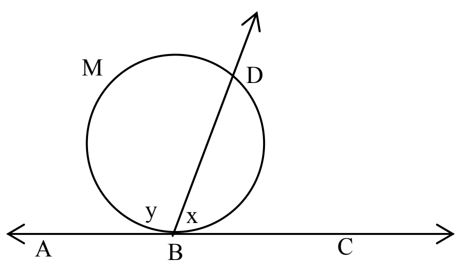
$\angle x$ is an angle formed by two secants whose vertex is outside the circle. Thus

$$m\angle x = \tfrac{1}{2}(m\widehat{PQ} - m\widehat{SR})$$
$$= \tfrac{1}{2}(106 - 38)$$
$$= \tfrac{1}{2}(68)$$
$$= 34$$

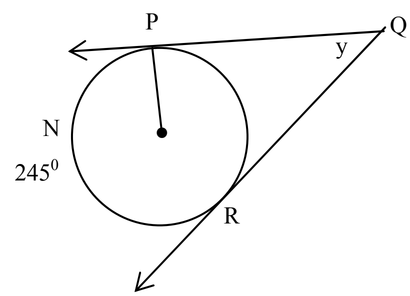
Example 2:

Find the unknown marked angles or arcs (x and y) in each figure:

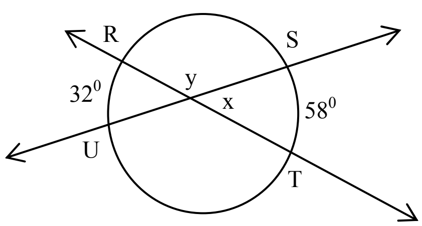
1.



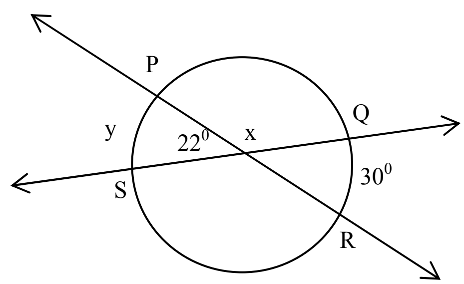
2.



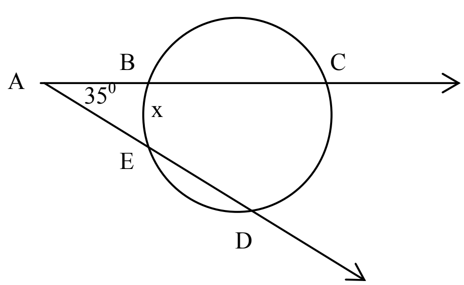
3.



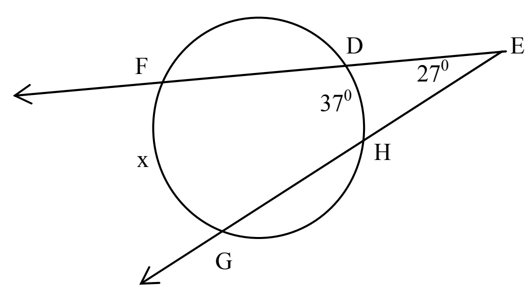
4.



5.



6.



Solutions:

1. Given : $m\overset{\frown}{BMD} = 210$

   Find: $m\angle x$, $m\angle y$

   $m\overset{\frown}{BMD} + m\overset{\frown}{BD} = 360$ (since the two arcs make the whole circle)

   $m\overset{\frown}{BD} = 360 - m\overset{\frown}{BMD}$

   $= 360 - 210$

   $= 150$

   a. $m\angle x = \frac{1}{2}\, m\overset{\frown}{BD}$

   $= \frac{1}{2}(150)$

   $= 75$

   b. $m\angle y = \frac{1}{2}\, m\overset{\frown}{BMD}$

   $= \frac{1}{2}(210)$

   $= 105$

2. Given: $m\overset{\frown}{PNR} = 245$

   Find: $m\angle x$, $m\angle y$

   $m\overset{\frown}{PNR} + m\overset{\frown}{PR} = 360$ (since the two arcs make a whole circle)

   $m\overset{\frown}{PR} = 360 - m\overset{\frown}{PNR}$

   $= 360 - 245$

   $= 115$

   But $m\angle x = m\overset{\frown}{PR}$ (Central angle equals numerically its intercepted arc)

   $m\angle x = 115$

   $m\angle y = \frac{1}{2}(m\overset{\frown}{PNR} - m\overset{\frown}{PR})$

   $= \frac{1}{2}(245 - 115)$

   $= \frac{1}{2}(130)$

   $= 65$

3. Given: $m\overset{\frown}{RU} = 32$, $m\overset{\frown}{ST} = 58$

   Find: $m\angle x$, $m\angle y$

   $m\angle x = \frac{1}{2}(m\overset{\frown}{RU} + m\overset{\frown}{ST})$

   $= \frac{1}{2}(32 + 58)$

   $= \frac{1}{2}(90)$

   $= 45$

$m\angle x + m\angle y = 180$ Linear Pair Postulate

$m\angle y = 180 - m\angle x$

$m\angle y = 180 - 45$

$= 135$

4. Given: $m\overparen{QR} = 30$, $m\angle 1 = 22$

   Find : $m\angle x$, y

   First, find y by using $m\angle 1$

   $m\angle 1 = \frac{1}{2}(y + 30)$

   $22 = \frac{1}{2}(y + 30)$

   $2(22) = y + 30$

   $44 = y + 30$

   $y = 44 - 30$

   $y = 14$

   To check;

   $m\angle 1 = \frac{1}{2}(m\overparen{PS} + m\overparen{QR})$

   $= \frac{1}{2}(14 + 30)$

   $= \frac{1}{2}(44)$

   $= 22$

5. Given: $m\angle A = 35$, $m\overparen{CD} = 110$

   Find: x

   $m\angle A = \frac{1}{2}(110 - x)$

   $35 = \frac{110 - x}{2}$

   $2(35) = 110 - x$

   $70 = 110 - x$

   $x = 110 - 70$

   $x = 40$

   To check:

   $m\angle A = \frac{1}{2}(110 - x)$

   $m\angle A = \frac{1}{2}(110 - 40)$

   $m\angle A = \frac{1}{2}(70)$

   $m\angle A = 35$

6. Given: $m\angle E = 27$, $m\overparen{DH} = 37$

   Find: x

   $m\angle E = \frac{1}{2}(x - 37)$

   $27 = \frac{x - 37}{2}$

   $54 = x - 37$

   $x = 54 + 37$

   $x = 91$

   To check:

   $m\angle E = \frac{1}{2}(x - 37)$

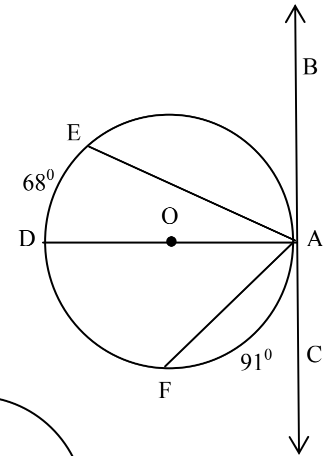
   $m\angle E = \frac{1}{2}(91 - 37)$

   $m\angle E = \frac{1}{2}(54)$

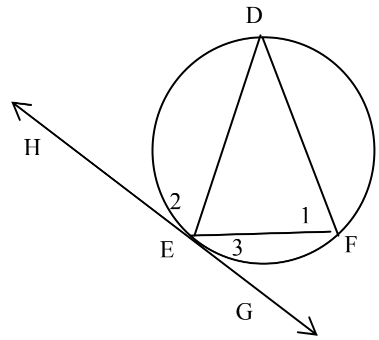
   $m\angle E = 27$

Try this out.

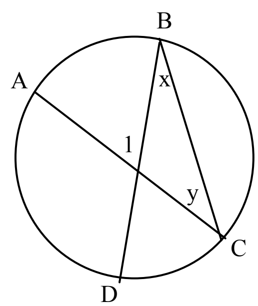
1. $\overleftrightarrow{BC}$ is tangent to circle O at A. $m\widehat{DE} = 68$
   $m\widehat{AF} = 91$. Find
   a. $m\widehat{EA}$
   b. $m\widehat{DF}$
   c. $m\angle BAE$
   d. $m\angle EAD$
   e. $m\angle DAF$
   f. $m\angle FAC$



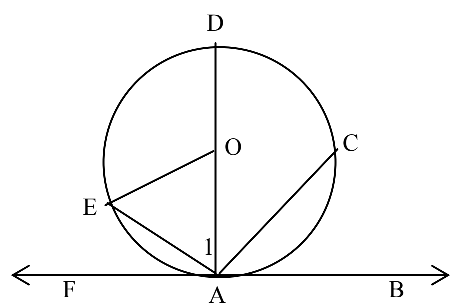
2. ΔDEF is isosceles with $\overline{DE} \cong \overline{DF}$.
   $m\angle 1 = 82$. Find
   a. $m\angle D$
   b. $m\widehat{DE}$
   c. $m\widehat{EF}$
   d. $m\angle 2$
   e. $m\angle 3$



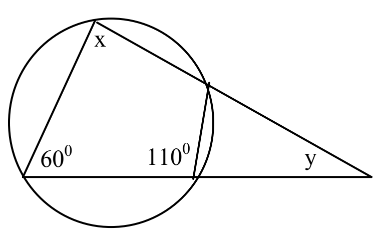
3. If $x = 18$ and $y = 23$,
   find $m\angle 1$.



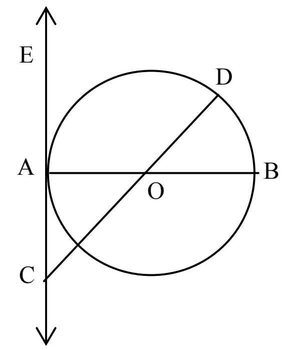
4. If $m\widehat{DE} = 108$ and $m\angle DOC = 85$, find:
   a. $m\widehat{EA}$
   b. $m\angle EAF$
   c. $m\angle DAC$
   d. $m\angle CAB$
   e. $m\angle 1$



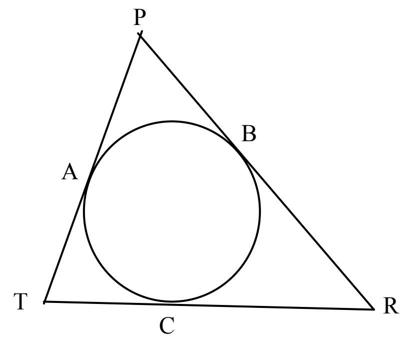
5. Using the given figure, find x and y.



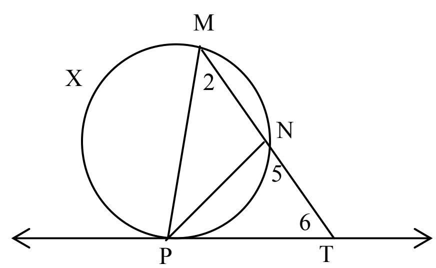
6. $\overleftrightarrow{EC}$ is tangent to circle O. $\overline{AB}$ is a diameter.
   If $m\overset{\frown}{DB} = 47$, find $m\overset{\frown}{AD}$, $m\angle ECD$



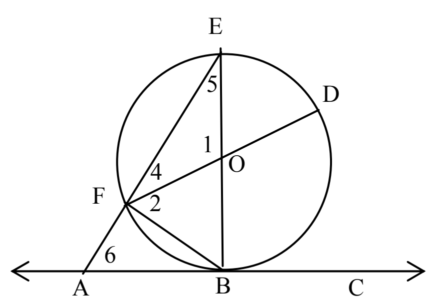
7. A polygon is said to be circumscribed about a circle if its sides are tangent to the circle.
   ΔPRT is circumscribed about circle O.
   If PT = 10, PR = 13 and RT = 9, find
   AP, TC and RB.



8. $\overleftrightarrow{PT}$ is tangent to circle O at P.
   If $m\overset{\frown}{NP} = 90$, and $m\overset{\frown}{MXP} = 186$, find
   a. $m\angle 1$
   b. $m\angle 2$
   c. $m\angle 3$
   d. $m\angle 4$
   e. $m\angle 5$
   f. $m\angle 6$



9. O is the center of the given circle.
   If $m\overset{\frown}{BD} = 122$ find
   a. $m\angle 1$
   b. $m\angle 2$
   c. $m\angle 3$
   d. $m\angle 4$
   e. $m\angle 5$
   f. $m\angle 6$



Let's Summarize

1. A tangent is a line that intersect a circle at only one point.

2. A. secant is a line that intersect a circle at two points.

3. If a line is tangent to a circle, it is perpendicular to a radius at the point of tangency.

4. If two tangents are drawn from an exterior point to a circle then

   a) the two tangent segments are congruent
   b) the angle between the segment and the line joining the external point and the center of the circle are congruent.

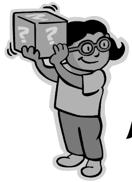
5. The measure of an angle formed by a secant and a tangent intersecting on the circle is equal to one half the measure of the intercepted arc.

6. The measure of an angle formed by two tangents from a common external point is equal to one-half the difference of the measures of the intercepted arcs.

7. The measure of an angle formed by secant and tangent intersecting outside the circle is equal to one-half the difference of the measures of the intercepted arcs.

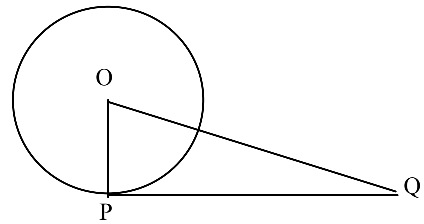
8. The measure of an angle formed by two secants intersecting inside the circle is equal to one-half the sum of the measure of the intercepted arc of the angle and its vertical angle pair.

9. The measure of an angle formed by two secants intersecting outside the circle is equal to one-half the difference of the measures of the intercepted arcs.

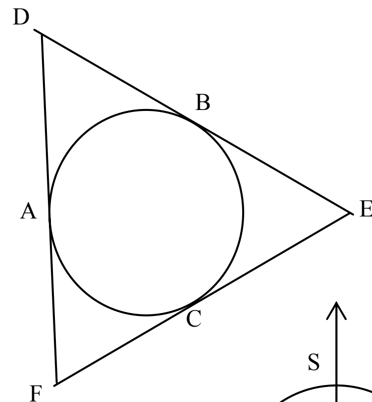


What have you learned

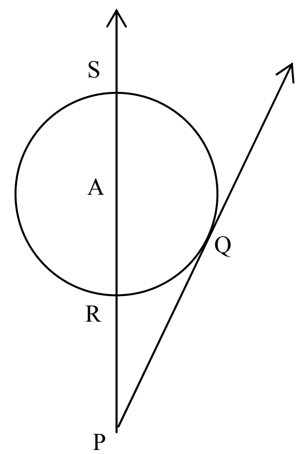
1. $\overline{QP}$ is tangent to circle O at P. If $m\angle POQ$ = 73, what is $m\angle Q$?



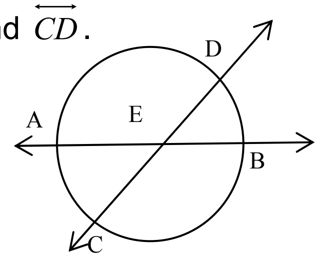
2. $\overline{DE}$, $\overline{EF}$ and $\overline{DF}$ are tangents to circle M. If DB =- 5, EC = 7 and AF = 4, what is the perimeter of ΔDEF?



3. $\overrightarrow{PS}$ AND $\overrightarrow{PQ}$ are secant and tangent of circle A. If $m\overset{\frown}{RQ}$ = 52, what is $m\angle P$?

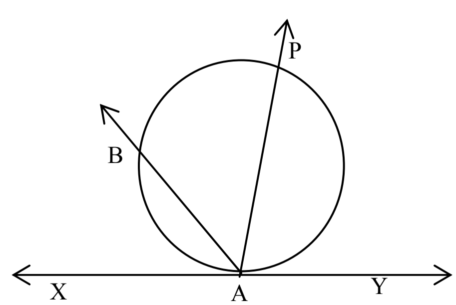


4. Given circle E with secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. If $m\overset{\frown}{BD}$ = 53 and $m\overset{\frown}{BC}$ = 117, find $m\angle BED$.

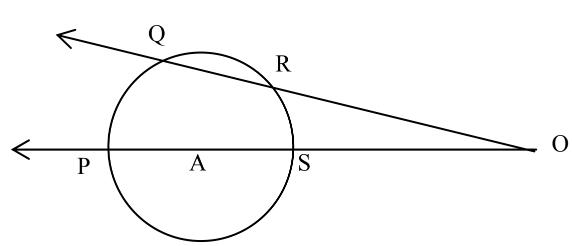


5. $\overleftrightarrow{XY}$ is tangent to circle at A . If $m\overset{\frown}{AB}$ = 105, and AB $\cong$ BP, find $m\angle BAP$ and

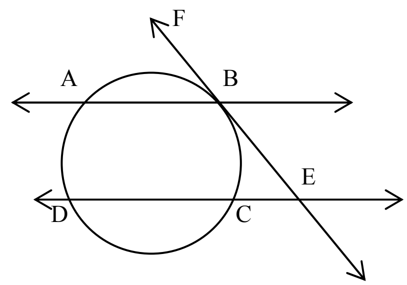
6. $m\angle PAY$



7. Given circle A with secants $\overrightarrow{OQ}$ and $\overrightarrow{OP}$.
If $m\overset{\frown}{RS}$ = 32 and $m\overset{\frown}{QR} = 2m\overset{\frown}{RS}$, find $m\angle O$?



8. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$. $\overleftrightarrow{BE}$ is tangent to the circle at B and intersects $\overleftrightarrow{DE}$ at E.
If $m\overset{\frown}{AB}$ = 110 and $m\overset{\frown}{AD}$ = 70, then $m\angle ABF$ = ____________.



9. $m\angle BEC$ = ___________.

10. Using the same figure, if $m\angle E$ = 42, $m\overset{\frown}{BC}$ = 60, find $m\overset{\frown}{AB}$.

Answer Key

How much do you know

1. $\cong$ or congruent
2. $\perp$ or perpendicular
3. 40
4. 22
5. 59
6. 61
7. 33
8. 120
9. 20
10. 18

Try this out

Lesson 1

1. a. 112
   b. 89
   c. 56
   d. 34
   e. 44.5
   f. 45.5

2. a. 16
   b. 164
   c. 32
   d. 82
   e. 16

3. 41
4. a. 72
   b. 36
   c. 42.5
   d. 47.5
   e. 54

5. x = 70
   y = 50

6. $m\widehat{AD} = 133$
   $m\angle ECD = 43$

7. AP = 7
   TC = 3
   RB = 6

8. a. 45
   b. 45
   c. 42
   d. 93
   e. 57
   f. 48

9. a. 122
   b. 61
   c. 29
   d. 29
   e. 29
   f. 45

What have you learned

1. 17
2. 32
3. 38
4. 58
5. 52.5
6. 75
7. 26
8. 55
9. 55
10. 84