Module 3
Triangle Congruence

What this module is about

This module is about using triangle congruence to prove congruent segments and angles. You will understand that a correspondence between two triangles is a congruence if the corresponding angle and corresponding sides are congruent.

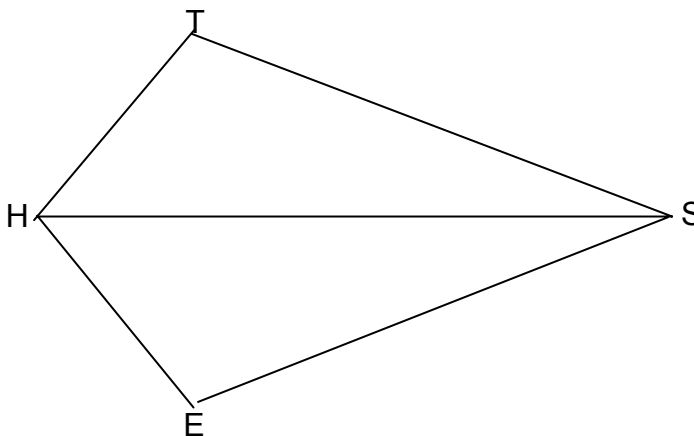## What you are expected to learn

This module is designed for you to:

1. prove congruent segments and angles using the conditions for triangle congruence.

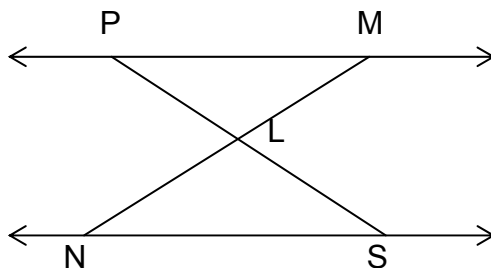2. solve routine and non-routine problems.

How much do you know

1. Given: $\overline{HS}$ bisects $\angle THE$
   $\angle HTS \cong \angle HES$

   Prove: $\overline{TS} \cong \overline{ES}$
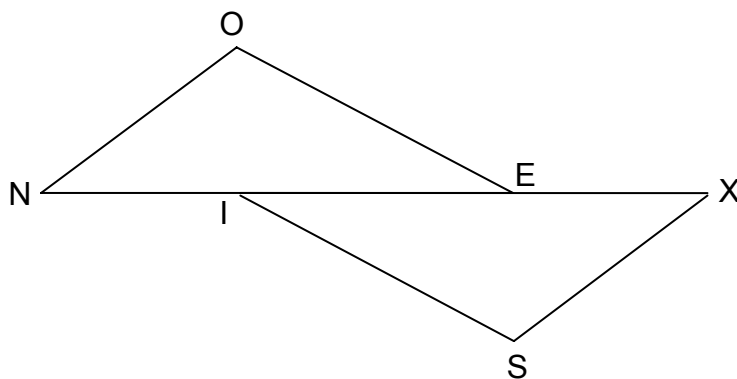
2. Given: $\overline{PM}$ // $\overline{NS}$; $\overline{PM} \cong \overline{SN}$
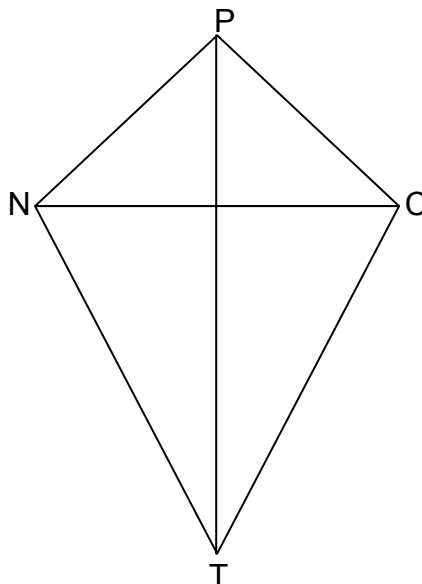
   Prove: $\overline{ML} \cong \overline{LN}$

3. Given: $\overline{NO} \cong \overline{XS}$; $\overline{OE} \cong \overline{SI}$; $\overline{NI} \cong \overline{XE}$
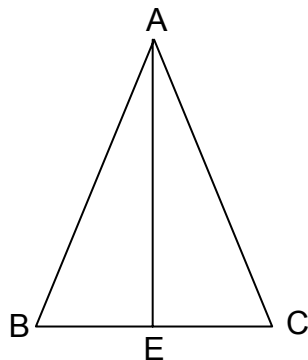
   Prove: $\angle O \cong \angle S$

4. Given: $\triangle PNC$ and $\triangle TNC$ are isosceles triangles with common base $\overline{NC}$.

   Prove: $\angle PNT \cong \angle PCT$

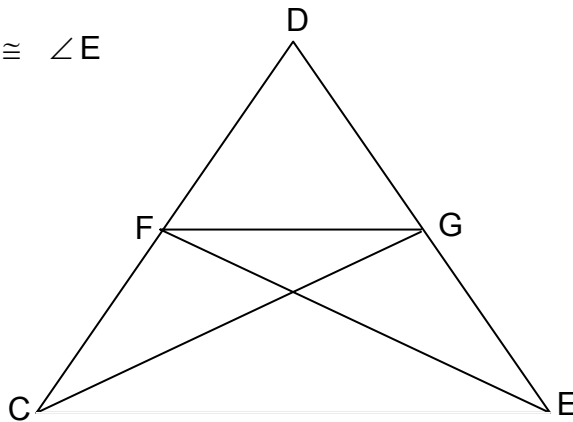5. Given: $\overline{AE}$ is the $\perp$ bisector of $\overline{BC}$
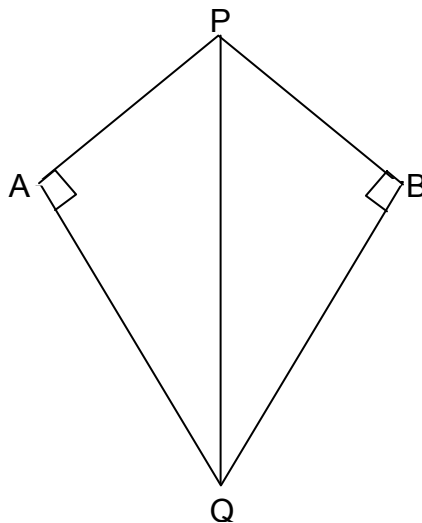
   Prove: $\angle B \cong \angle C$

6. Given: $\overline{CH} \cong \overline{EH}$, $\overline{FH} \cong \overline{GH}$

   Prove: $\angle C \cong \angle E$

7. Given: $\angle A$ and $\angle B$ are right angles and $\overline{AP} \cong \overline{BP}$.

   Prove: $\overline{AQ} \cong \overline{BQ}$

8. Given: $\angle 1 \cong \angle 2$
   O is the midpoint of $\overline{SP}$

   Prove: $\overline{MO} \cong \overline{NO}$
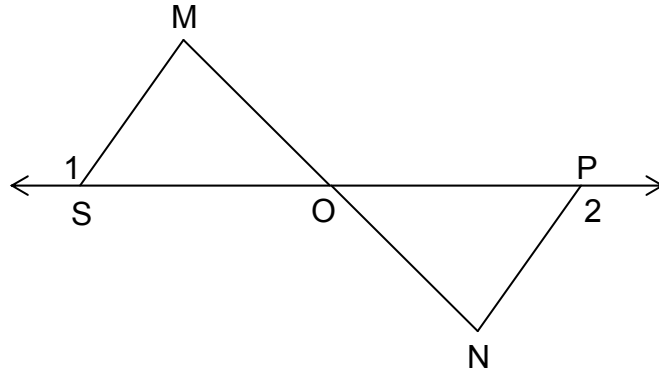
9. Given: $\overline{LO} \cong \overline{LN}$
   $\triangle JLO$ and $\triangle MLN$ are right $\triangle s$

   Prove: $\angle J \cong \angle K$

10. Given: $\overline{PQ} \cong \overline{RS}$
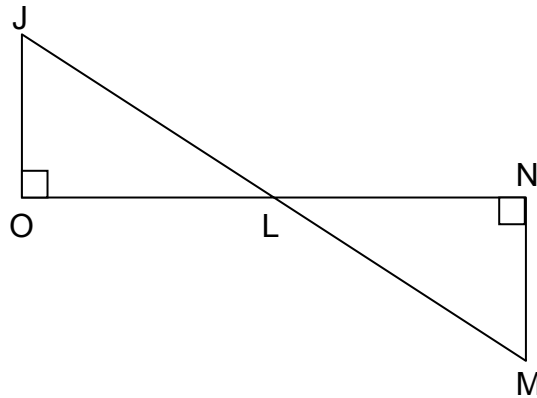 $\angle QRP \cong \angle SRP$

    Prove: $\overline{SP} \cong \overline{QR}$

4
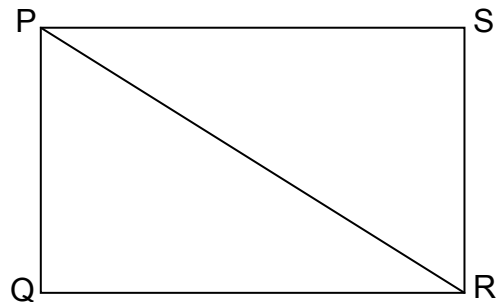
Lesson

Congruent Segments and Congruent Angles

To prove two segments or two angles are congruent, you must show that they are corresponding parts of congruent triangles.

For triangle congruence, you have the following:

- SSS congruence
- SAS congruence
- ASA congruence
- SAA congruence

For right triangle congruence, you have the following:

- LL congruence
- LA congruence
- HyL congruence
- HyA congruence

Examples:

Formal Proofs:

1. Given: $\overline{AB}$ // $\overline{DC}$, $\overline{AD}$ // $\overline{BC}$

    Prove: $\overline{AB} \cong \overline{DC}$

Proof:

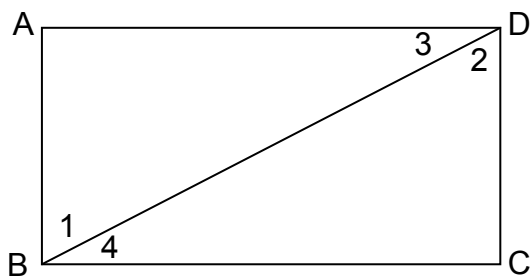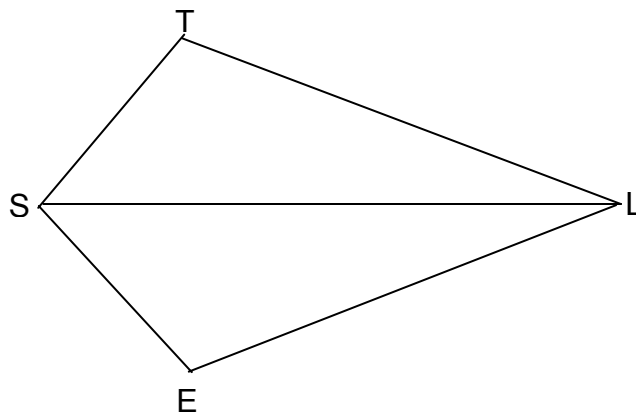| Statement	Reason
1. $\overline{AB}$ // $\overline{DC}$, $\overline{AD}$ // $\overline{BC}$	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. If two parallel lines are cut by a transversal, then the alternate interior angles are $\cong$
3. $\overline{BD} \cong \overline{BD}$	3. Reflexivity
4. $\triangle ABD \cong \triangle CDB$	4. ASA
5. $\therefore \overline{AB} \cong \overline{DC}$	5. Corresponding parts of $\cong$ triangles are $\cong$ or CPCTC

2. Given: $\overline{LS}$ bisects $\angle TLE$
              $\angle LTS \cong \angle LES$

   Prove: $\overline{TS} \cong \overline{ES}$

Proof:

Statement	Reason
1. $\overline{LS}$ bisects $\angle TLE$	1. Given
2. $\angle TLS \cong \angle ELS$	2. Definition of angle bisector
3. $\overline{LS} \cong \overline{LS}$	3. Reflexivity
4. $\angle LTS \cong \angle LES$	4. Given
5. $\triangle TSL \cong \triangle ESL$	5. SAA congruence
6. $\overline{TS} \cong \overline{ES}$	6. Corresponding parts of $\cong$ $\triangle$s are $\cong$ or CPCTC

3. Given: $\overline{MO} \cong \overline{YS}$, $\overline{OB} \cong \overline{SP}$, $\overline{MP} \cong \overline{YB}$

Prove: $\angle O \cong \angle S$

O

B

M P Y

S
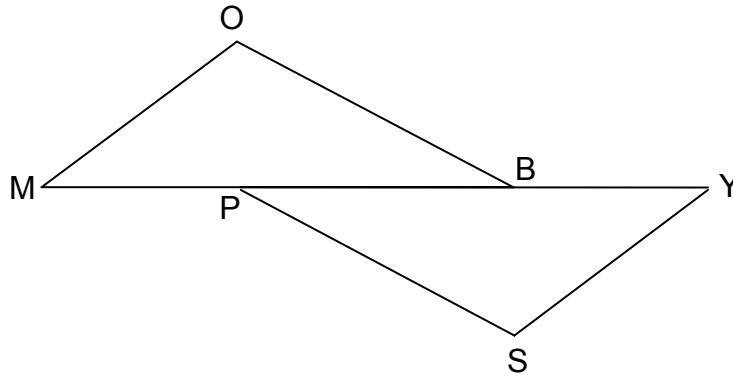
Proof:

Statement	Reason
1. $\overline{MO} \cong \overline{YS}$ $\overline{OB} \cong \overline{SP}$ $\overline{MP} \cong \overline{YB}$	1. Given
2. $\overline{PB} \cong \overline{PB}$	2. Reflexivity
3. $\overline{MB} \cong \overline{PY}$	3. By addition
4. $\triangle MOB \cong \triangle YSP$	4. SSS congruence
5. $\angle O \cong \angle S$	5. CPCTC

4. Given: $\overline{AR}$ is the $\perp$ bisector of $\overline{BX}$.

Prove: $\angle B \cong \angle X$

A

B X
R
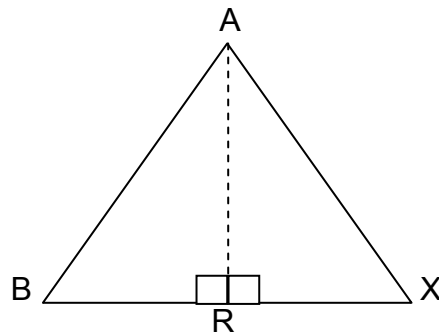
Proof:

Statement	Reason
1. $\overline{AR}$ is the $\perp$ bisector of $\overline{BX}$	1. Given
2. $\overline{AR} \cong \overline{AR}$	2. Reflexivity
3. $\overline{BR} \cong \overline{XR}$	3. Definition of $\perp$ bisector
4. $\triangle ARB \cong \triangle ARX$	4. LL congruence
5. $\angle B \cong \angle X$	5. CPCTC

5. Given: $\angle 1 \cong \angle 2$
   O is the midpoint of SP

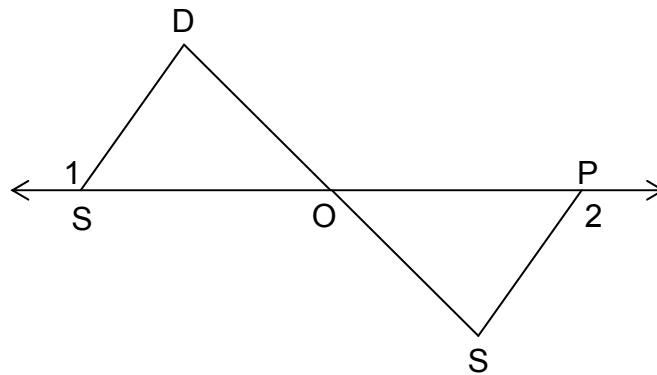Prove: $\overline{DO} \cong \overline{SO}$

Proof:

Statement	Reason
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle DSO \cong \angle SPO$	2. Supplements of $\cong$ $\angle$s are also $\cong$
3. O is the midpoint of SP	3. Given
4. $\overline{SO} \cong \overline{PO}$	4. Definition of midpoint
5. $\angle DOS \cong \angle SOP$	5. Vertical $\angle$s are $\cong$
6. $\triangle SDO \cong \triangle PSO$	6. ASA congruence
7. $\overline{DO} \cong \overline{SO}$	7. CPCTC

Try this out

1. Given: ∠SLO and ∠KMO are right angles
          $\overline{LO} \cong \overline{MO}$

   Prove: ∠S ≅ ∠K
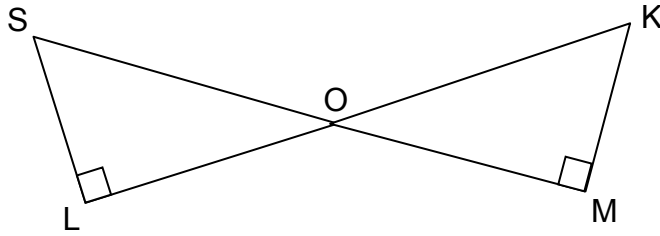
2. Given: $\overline{XY} \cong \overline{RS}$
          ∠YXR ∠SRX

   Prove: $\overline{SX} \cong \overline{YR}$
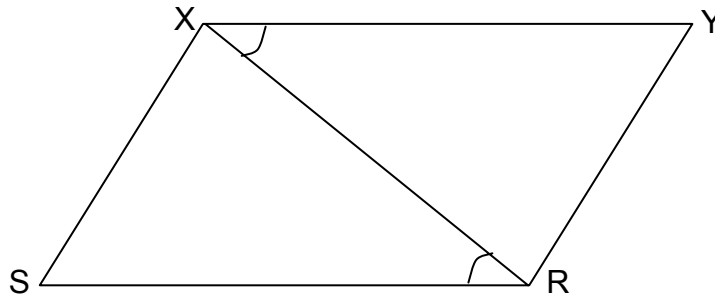
3. Given: △PNU and △TNU are isosceles triangles with common base $\overline{NU}$

   Prove: ∠PNT ≅ ∠PUT
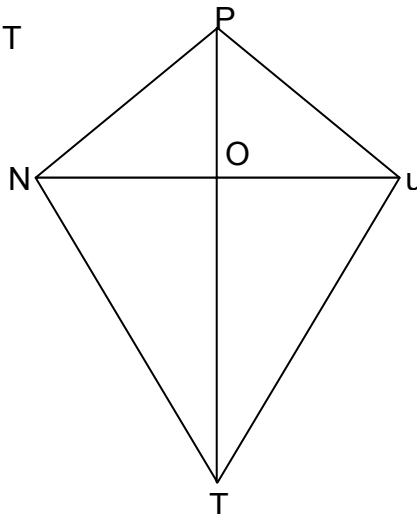
4. Given: $\overline{CD} \cong \overline{ED}$, $\overline{FC} \cong \overline{FE}$

   Prove: $\angle C \cong \angle E$

5. Given: $\angle A \cong \angle C$ are right angles
   $\overline{AK} \cong \overline{MC}$

   Prove: $\overline{MA} \cong \overline{KC}$
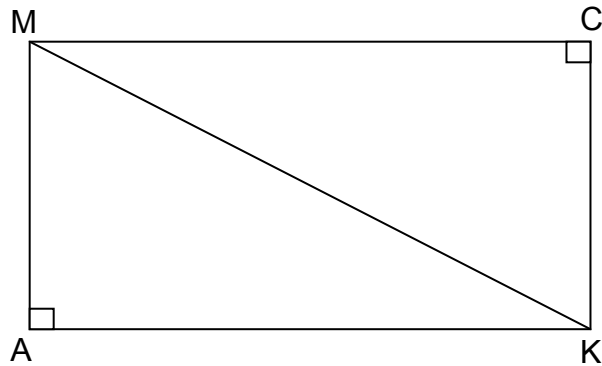
6. Given: $\overline{MN} \perp \overline{NR}$, $\overline{PR} \perp \overline{NR}$, $\overline{MR} \cong \overline{PN}$

   Prove: $\overline{MN} \cong \overline{NR}$
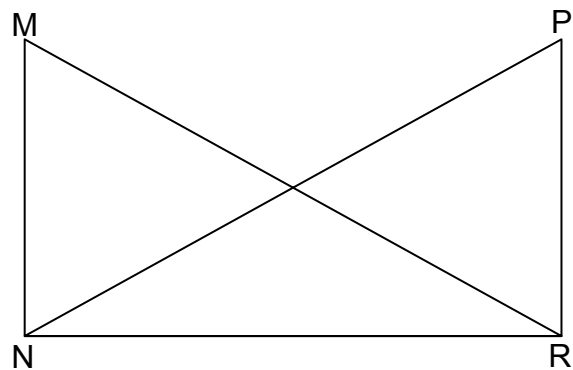
7. Given: $\overline{AI} \cong \overline{BN}, \overline{BI} \cong \overline{AN}$

   Prove: $\angle I \cong \angle N$
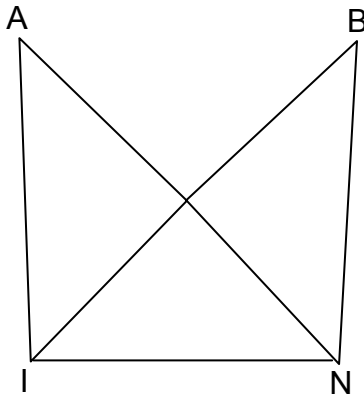
8. Given: In the figure, $\overline{JV}$ and $\overline{NC}$ bisect each other at O

   Prove: $\angle J \cong \angle V$
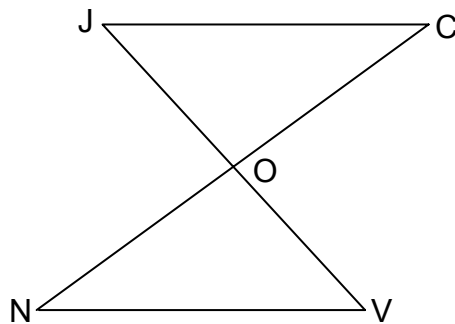
9. Given: $\overline{MQ}$ and $\overline{PN}$ bisect each other at O

   Prove: $\angle P \cong \angle N$
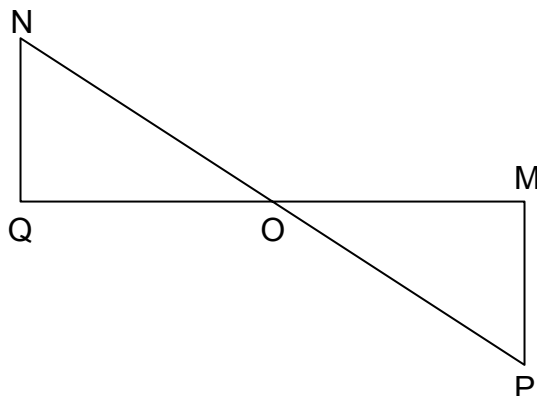
10. Given: $\overline{GQ} \cong \overline{RS}$
$\angle QGR \cong \angle SRG$

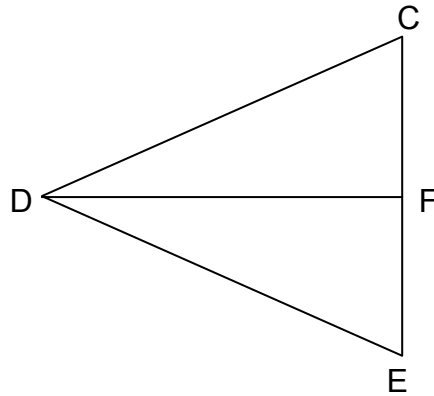Prove: $\overline{SG} \cong \overline{QR}$
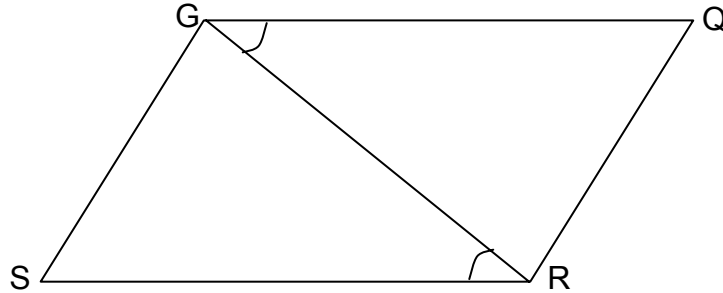
 Let's summarize

       To prove two segments or two angles are congruent, you must show that they are corresponding parts of congruent triangles.

       For triangle congruence, you have the following:

- SSS congruence
- SAS congruence
- ASA congruence
- SAA congruence

       For right triangle congruence, you have the following:

- LL congruence
- LA congruence
- HyL congruence
- HyA congruence

1. Given: $\overline{TS}$ bisects $\angle MTE$
            $\angle TMS \cong \angle TES$

   Prove: $\overline{TM} \cong \overline{TE}$
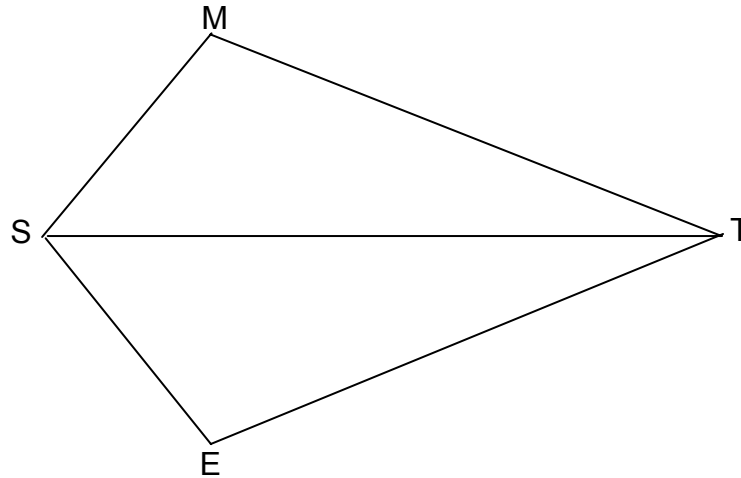
2. Given: $\overline{PQ} // \overline{NS}$; $\overline{PQ} \cong \overline{SN}$

   Prove: $\overline{QL} \cong \overline{LN}$
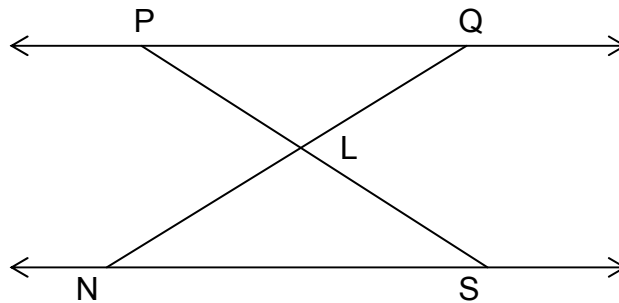
3. Given: $\overline{XO} \cong \overline{YS}$; $\overline{OE} \cong \overline{SI}$; $\overline{XI} \cong \overline{YE}$

Prove: $\angle O \cong \angle S$
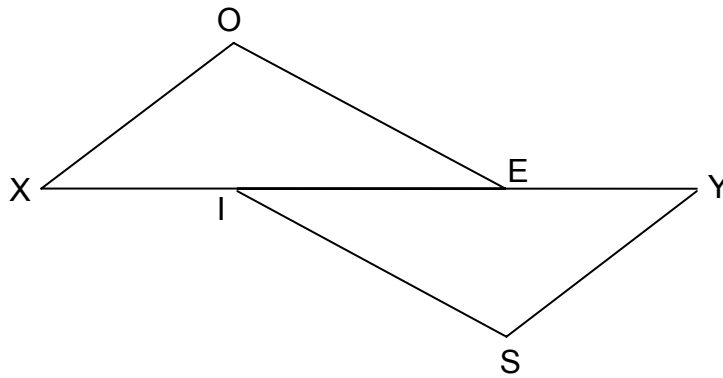
4. Given: $\triangle PNQ$ and $\triangle TNQ$ are isosceles triangles with common base $\overline{NQ}$.

Prove: $\angle PNQ \cong \angle TNQ$
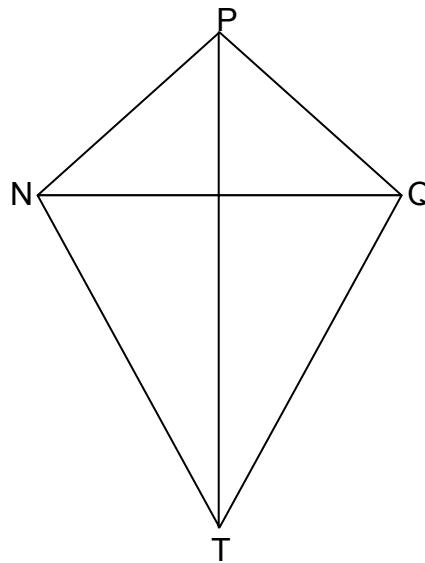
5. Given: $\overline{HE}$ is the $\perp$ bisector of $\overline{BC}$

Prove: $\angle B \cong \angle C$
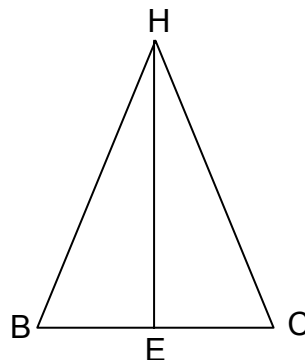
6. Given: $\overline{CG} \cong \overline{EG}$, $\overline{FG} \cong \overline{HG}$

Prove: $\angle C \cong \angle E$
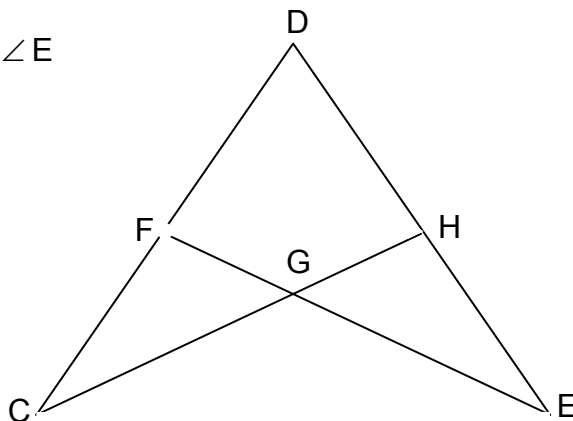
7. Given: $\angle R$ and $\angle S$ are right angles and $\overline{RP} \cong \overline{SP}$.

Prove: $\overline{RQ} \cong \overline{SQ}$
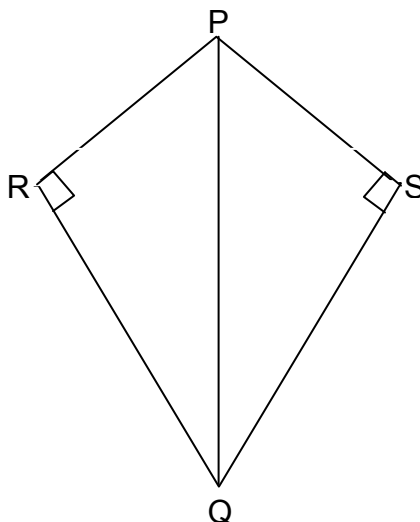
8. Given: $\angle 1 \cong \angle 2$

O is the midpoint of $\overline{SP}$

Prove: $\overline{RO} \cong \overline{TO}$
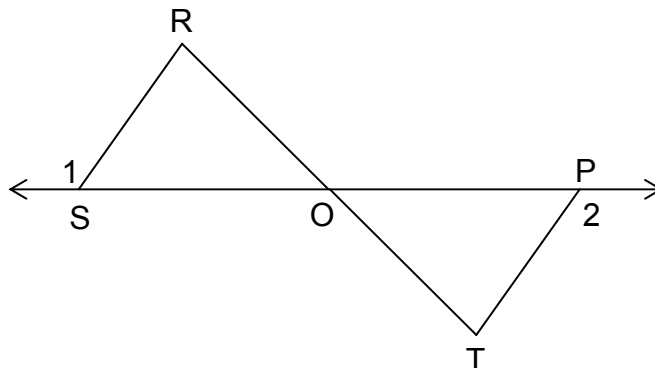
9. Given: ∠JLO and ∠KMO are right ∠s
          LO and MO

    Prove: ∠J ≅ ∠K

J   K

                   O

L   M
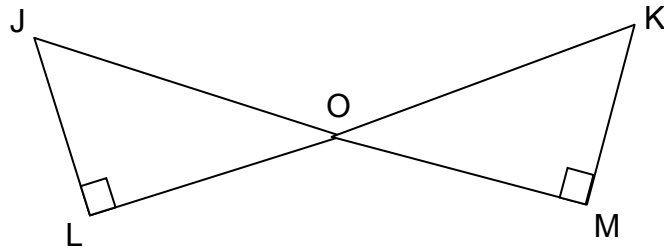
10. Given: $\overline{PQ} \cong \overline{RS}$
           ∠QPR ≅ ∠SRP

    Prove: $\overline{SP} \cong \overline{QR}$

P   Q

S   R

Answer key
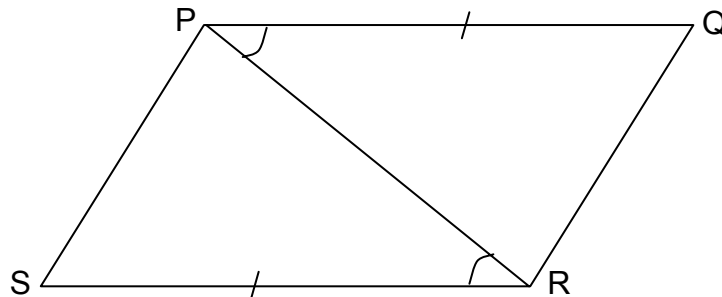
How much do you know

1. Proof

Statement	Reason
1. $\overline{HS}$ bisects $\angle THE$ $\angle HTS \cong \angle HES$	1. Given
2. $\angle THS \cong \angle EHS$	2. Definition of $\angle$ bisector
3. $\overline{HS} \cong \overline{HS}$	3. Reflexivity
4. $\triangle THS \cong \triangle EHS$	4. SAA congruence
5. $\overline{TS} \cong \overline{ES}$	5. CPCTC

2. Proof:

Statement	Reason
1. $\overline{PM} \parallel \overline{NS}, \overline{PM} \cong \overline{SN}$	1. Given
2. $\angle P \cong \angle S, \angle M \cong \angle N$	2. If 2 // lines cut by a transversal, the alternate interior $\angle$s are $\cong$
3. $\triangle PLM \cong \triangle SLN$	3. ASA congruence
4. $\overline{ML} \cong \overline{LN}$	4. CPCTC

3. Proof:

Statement	Reason
1. $\overline{NO} \cong \overline{XS}, \overline{OE} \cong \overline{SI}$ $\overline{NI} \cong \overline{XE}$	1. Given
2. $\overline{IE} \cong \overline{IE}$	2. Reflexivity
3. $\overline{NE} \cong \overline{IX}$	3. By addition
4. $\triangle NOE \cong \triangle XSI$	4. SSS congruence
5. $\angle O \cong \angle S$	5. CPCTC

4. Proof:

Statement	Reason
1. $\triangle PNC$ and $\triangle TNC$ are isosceles $\triangle$	1. Given
2. $\overline{PN} \cong \overline{PC}, \overline{TN} \cong \overline{TC}$	2. Definition of isosceles
3. $\overline{PT} \cong \overline{PT}$	3. Reflexivity
4. $\triangle PNT \cong \triangle PCT$	4. SSS congruence
5. $\angle PNT \cong \angle PCT$	5. CPCTC

5. Proof:

Statement	Reason
1. $\overline{AE}$ is the $\perp$ bisector of $\overline{BC}$	1. Given
2. $\overline{BE} \cong \overline{CE}$	3. Definition of $\perp$ bisector
3. $\overline{AE} \cong \overline{AE}$	2. Reflexivity
4. $\triangle AEB \cong \triangle AEC$	4. LL congruence
5. $\angle B \cong \angle C$	5. CPCTC

6. Proof:

Statement	Reason
1. $\overline{CH} \cong \overline{EH}$, $\overline{FH} \cong \overline{GH}$	1. Given
2. $\angle FHC \cong \angle GHE$	2. Vertical $\angle$s are $\cong$
3. $\triangle FHC \cong \triangle GHE$	3. SAS
4. $\angle C \cong \angle E$	4. CPCTC

7. Proof:

Statement	Reason
1. $\angle A$ and $\angle B$ are rt. $\angle$s $\overline{AP} \cong \overline{BP}$	1. Given
2. $\overline{PQ} \cong \overline{PQ}$	2. Reflexivity
3. $\triangle PAQ \cong \triangle PBQ$	3. HyL congruence
4. $\overline{AQ} \cong \overline{BQ}$	4. CPCTC

8. Proof:

Statement	Reason
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle MSO \cong \angle NPO$	2. Supplements of $\cong$ $\angle$s are also $\cong$
3. O is the midpoint of $\overline{SP}$	3. Given
4. $\overline{SO} \cong \overline{PO}$	4. Definition of midpoint
5. $\angle MOS \cong \angle NOP$	5. Vertical $\angle$s are $\cong$
6. $\triangle MDO \cong \triangle NPO$	6. ASA congruence
7. $\overline{MO} \cong \overline{NO}$	7. CPCTC

9. Proof:

Statement	Reason
1. $\overline{LO} \cong \overline{LN}$	1. Given
2. $\angle JLO \cong \angle MLN$	2. Vertical $\angle$s are $\cong$
3. $\triangle JLO \cong \triangle MLN$	3. LA congruence
4. $\angle J \cong \angle K$	4. CPCTC

10. Proof:

Statement	Reason
1. $\overline{PQ} \cong \overline{RS}$	1. Given
$\angle QRP \cong \angle SRP$
2. $\overline{PR} \cong \overline{PR}$	2. Reflexivity
3. $\triangle PSR \cong \triangle RQP$	3. SAS congruence
4. $\overline{SP} \cong \overline{QR}$	4. CPCTC

Try this out

Lesson

1. Proof:

Statement	Reason
1. $\overline{LO} \cong \overline{MO}$	1. Given
2. $\angle SOL \cong \angle KOM$	2. Vertical $\angle$s are $\cong$
3. $\triangle SLO \cong \triangle KMO$	3. LA congruence
4. $\angle S \cong \angle K$	4. CPCTC

2. Proof:

Statement	Reason
1. $\overline{XY} \cong \overline{RS}$	1. Given
$\angle YXR \cong \angle SRX$
2. $\overline{RX} \cong \overline{RX}$	2. Reflexivity
3. $\triangle XYR \cong \triangle RSX$	3. SAS congruence
4. $\overline{SX} \cong \overline{YR}$	4. CPCTC

3. Proof:

Statement	Reason
1. $\triangle PNU$ and $\triangle TNU$ are isosceles $\triangle$	1. Given
2. $\overline{PN} \cong \overline{PU}$, $\overline{TN} \cong \overline{TU}$	2. Definition of isosceles
3. $\overline{PT} \cong \overline{PT}$	3. Reflexivity
4. $\triangle PNT \cong \triangle PUT$	4. SSS congruence
5. $\angle PNT \cong \angle PUT$	5. CPCTC

4. Proof:

Statement	Reason
1. $\overline{CD} \cong \overline{ED}$, $\overline{FC} \cong \overline{FE}$	1. Given
2. $\overline{DF} \cong \overline{DF}$	2. Reflexivity
3. $\triangle DCF \cong \triangle DEF$	3. SSS congruence
4. $\angle C \cong \angle E$	4. CPCTC

5. Proof:

Statement	Reason
1. $\angle A \cong \angle C$ $\overline{AK} \cong \overline{MC}$	1. Given
2. $\overline{MK} \cong \overline{MK}$	2. Reflexivity
3. $\triangle MCK \cong \triangle KAM$	3. HyL congruence
4. $\overline{MA} \cong \overline{KC}$	4. CPCTC

6. Proof:

Statement	Reason
1. $\overline{MR} \cong \overline{PN}$	1. Given
2. $\overline{NR} \cong \overline{NR}$	2. Reflexivity
3. $\triangle MNR \cong \triangle PRN$	3. Hyl congruence
4. $\overline{MN} \cong \overline{PR}$	4. CPCTC

7. Proof:

Statement	Reason
1. $\overline{AI} \cong \overline{BN}$ $\overline{BI} \cong \overline{AN}$	1. Given
2. $\overline{IN} \cong \overline{IN}$	2. Reflexivity
3. $\triangle AIN \cong \triangle BNI$	3. SSS congruence
4. $\angle I \cong \angle N$	4. CPCTC

8. Proof:

Statement	Reason
1. $\overline{JV}$ and $\overline{NC}$ bisect each other at O	1. Given
2. $\overline{JO} \cong \overline{VO}$ $\overline{CO} \cong \overline{NO}$	2. Definition of Segment Bisector
3. $\angle JOC \cong \angle VON$	3. Vertical $\angle$s are $\cong$
4. $\triangle JOC \cong \triangle VON$	4. SAS
5. $\angle J \cong \angle V$	5. CPCTC

9. Proof:

Statement	Reason
1. $\overline{MQ}$ and $\overline{PN}$ bisect each other at O	1. Given
2. $\overline{MO} \cong \overline{QO}$ $\overline{PO} \cong \overline{NO}$	2. Definition of Segment Bisector
3. $\angle POM \cong \angle NOQ$	3. Vertical $\angle$s are $\cong$
4. $\triangle POM \cong \triangle VOQ$	4. SAS
5. $\angle P \cong \angle N$	5. CPCTC

10. Proof:

Statement	Reason
1. $\overline{GQ} \cong \overline{RS}$ $\angle QGR \cong \angle SRG$	1. Given
2. $\overline{RG} \cong \overline{RG}$	2. Reflexivity
3. $\triangle QGR \cong \triangle SRG$	3. SAS congruence
4. $\overline{SG} \cong \overline{QR}$	4. CPCTC

What have you learned

1. Proof

Statement	Reason
1. $\overline{TS}$ bisects $\angle MHE$ $\angle TMS \cong \angle TES$	1. Given
2. $\angle MTS \cong \angle ETS$	2. Definition of $\angle$ bisector
3. $\overline{TS} \cong \overline{TS}$	3. Reflexivity
4. $\triangle SMT \cong \triangle SET$	4. SAA congruence
5. $\overline{TM} \cong \overline{TE}$	5. CPCTC

2. Proof:

Statement	Reason
1. $\overline{PQ} \parallel \overline{SN}$	1. Given
2. $\angle QPL \cong \angle NSL$ $\angle PQL \cong \angle SNL$	2. If 2 $\parallel$ lines cut by a transversal, the alternate interior $\angle$s are $\cong$
3. $\triangle PQL \cong \triangle SNL$	3. ASA congruence
4. $\overline{QL} \cong \overline{LN}$	4. CPCTC

3. Proof:

Statement	Reason
1. $\overline{XO} \cong \overline{YS}, \overline{OE} \cong \overline{SI}$ $\overline{XI} \cong \overline{YE}$	1. Given
2. $\overline{IE} \cong \overline{IE}$	2. Reflexivity
3. $\overline{XE} \cong \overline{YI}$	3. By addition
4. $\triangle XOE \cong \triangle YSI$	4. SSS congruence
5. $\angle O \cong \angle S$	5. CPCTC

4. Proof:

Statement	Reason
1. $\triangle PNQ$ and $\triangle TNQ$ are isosceles $\triangle$	1. Given
2. $\overline{PN} \cong \overline{PQ}, \overline{TN} \cong \overline{TQ}$	2. Definition of isosceles
3. $\overline{PT} \cong \overline{PT}$	3. Reflexivity
4. $\triangle PNT \cong \triangle PQT$	4. SSS congruence
5. $\angle PNQ \cong \angle TNQ$	5. CPCTC

5. Proof:

Statement	Reason

1. $\overline{HE}$ is the $\perp$ bisector of $\overline{BC}$ 1. Given
2. $\overline{BE} \cong \overline{CE}$ 3. Definition of $\perp$ bisector
3. $\overline{HE} \cong \overline{HE}$ 2. Reflexivity
4. $\triangle HEB \cong \triangle HEC$ 4. LL congruence
5. $\angle B \cong \angle C$ 5. CPCTC

6. Proof:

Statement	Reason

1. $\overline{CG} \cong \overline{EG}$, $\overline{FG} \cong \overline{HG}$ 1. Given
2. $\angle FGC \cong \angle HGE$ 2. Vertical $\angle$s are $\cong$
3. $\triangle FGC \cong \triangle HGE$ 3. SAS
4. $\angle C \cong \angle E$ 4. CPCTC

7. Proof:

Statement	Reason

1. $\overline{RP} \cong \overline{SP}$ 1. Given
2. $\overline{PQ} \cong \overline{PQ}$ 2. Reflexivity
3. $\triangle PRQ \cong \triangle PSQ$ 3. HyL congruence
4. $\overline{RQ} \cong \overline{SQ}$ 4. CPCTC

8. Proof:

Statement	Reason

1. $\angle 1 \cong \angle 2$ 1. Given
2. $\angle RSP \cong \angle TPO$ 2. Supplements of $\cong \angle$s are also $\cong$
3. O is the midpoint of $\overline{SP}$ 3. Given
4. $\overline{SO} \cong \overline{PO}$ 4. Definition of midpoint
5. $\angle SOR \cong \angle POT$ 5. Vertical $\angle$s are $\cong$
6. $\triangle SOR \cong \triangle POT$ 6. ASA congruence
7. $\overline{RO} \cong \overline{TO}$ 7. CPCTC

9. Proof:

Statement

1. $\overline{LO} \cong \overline{MO}$
2. $\angle LOJ \cong \angle MOK$
3. $\triangle LOJ \cong \triangle MOK$
4. $\angle J \cong \angle K$

Reason

1. Given
2. Vertical $\angle$s are $\cong$
3. LA congruence
4. CPCTC

10. Proof:

Statement

1. $\overline{PQ} \cong \overline{RS}$
   $\angle QPR \cong \angle SRP$
2. $\overline{PR} \cong \overline{PR}$
3. $\triangle QPR \cong \triangle SRP$
4. $\overline{SP} \cong \overline{QR}$

Reason

1. Given

2. Reflexivity
3. SAS congruence
4. CPCTC