

Module 5

Geometry of Shape and Size



What this module is about

This module will discuss the perimeter of the commonly used polygons in geometry such as triangles, quadrilaterals and others (with their corresponding formulas). In addition, this module will also expound on the circumference of the circle. Furthermore, this module will help you apply these concepts in solving problems associated with real life.



What you are expected to learn

This module is written for you to

1. Recall the different plane figures and their properties which are commonly used in geometry.
2. Name the properties of the sides of different polygons.
3. Define perimeter and determine the different formulas of getting the perimeter of the different polygons .
4. Define a circle.
5. Identify the lines and segments associated with circles.
6. Define circumference of a circle and determine the formula for getting the circumference of the circle.
7. Use the formulas for finding perimeter and circumference in solving real life problems.



How much do you know

Answer the following questions as indicated.

Find the perimeter of a regular polygon(indicated) given the length of a side.

1. triangle, $s = 13 \text{ cm}$
2. square, $s = 10 \text{ cm}$
3. rhombus, $s = 11.5 \text{ dm}$
4. pentagon, $s = 9.25 \text{ cm}$
5. hexagon, $s = 12.73 \text{ cm}$

6. The circumference of a circle is 66 cm. Find the radius of the circle. Use $\pi = \frac{22}{7}$.
7. The length of a rectangle is 13 more than twice its width. If the perimeter is 116 cm, find the dimensions of the rectangle.
8. One of the sides of a rhombus is $2x + 1$. What is its perimeter?
9. The diameter of a circle is 14 cm. Find its circumference. Use $\pi = 3.14$.
10. If the perimeter of a regular hexagon is 69 cm, what is the length of each side?



What you will do

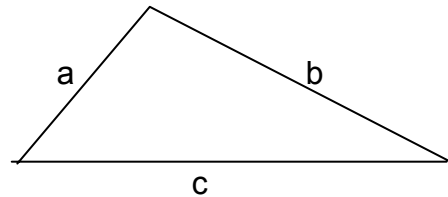
Lesson 1

Perimeters of Polygon

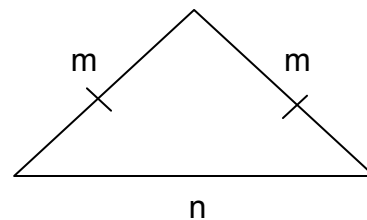
The perimeter of a polygon is the distance around it. It can be computed by getting the sum of the length or measures of all the sides. If the polygon is identified as regular, then the perimeter is computed by simply multiplying the given measure of the side with the number of sides. Let us recall the different formulas for finding the perimeter of different polygons.

Triangle:

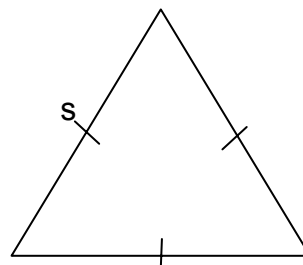
For a general triangle, the perimeter (P) is
 $P = a + b + c$, where a, b, and c
 are the measures of the sides.



For isosceles triangle, perimeter is
 $P = m + m + n$, where m is the
 length of one of the legs, and
 n is the length of the base.

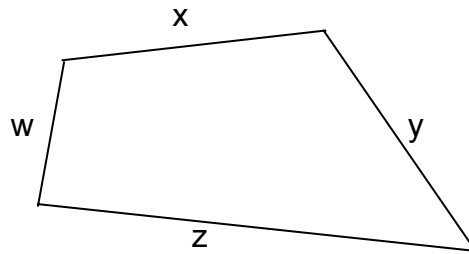


For equilateral triangle, the perimeter is
 $P = 3s$, where s is the length of
 one of the equal sides.

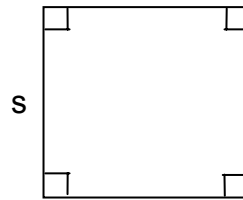


For polygon of four sides or quadrilaterals, consider the given figures.

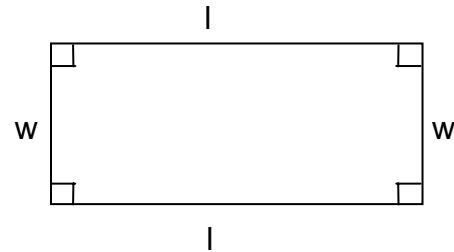
Given quad ABCD, then its perimeter is
 $P = w + x + y + z$, where w , x , y
 and z are the length of the sides



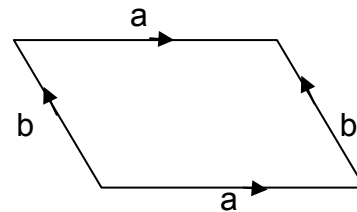
For a square, its perimeter is
 $P = s + s + s + s$, where s is the
 length of a side. Therefore,
 $P = 4s$



For a rectangle, the perimeter is
 $P = l + l + w + w$, where l is the
 length and w is the width. Therefore,
 $P = 2l + 2w$ or
 $P = 2(l + w)$

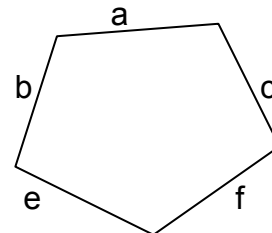


For a parallelogram, the perimeter is
 $P = a + a + b + b$, where a , and b are the
 lengths of the two consecutive sides
 of the parallelogram. Thus
 $P = 2a + 2b$
 $P = 2(a + b)$



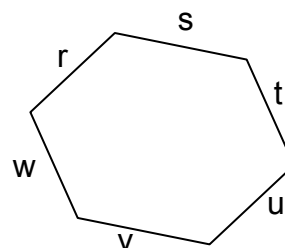
For other figures like pentagon and hexagon or those polygons with more than five sides, the formula is practically the same. Get the sum of all the length of the sides.

For the given pentagon, the perimeter is
 $P = a + b + c + d + e$, where a , b , c ,
 d , and e are the measures of the
 sides.



For a regular pentagon with a as the length of a side, the perimeter is
 $P = 5a$

For a hexagon, the perimeter is
 $P = r + s + t + u + v + w$, where
 r , s , t , u , v , and w are the length
 of the sides.

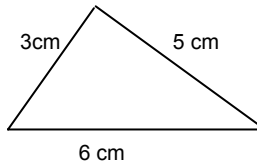


For a regular hexagon whose length of side is denoted by r , the perimeter is given by the formula

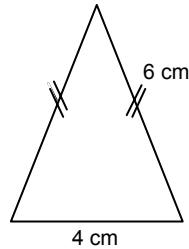
$$P = 6r$$

Example 1. Find the perimeter of the following figures.

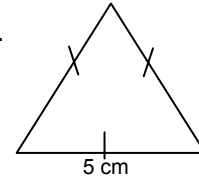
1.



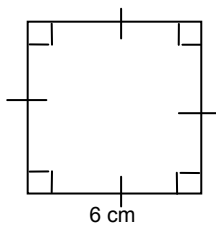
2.



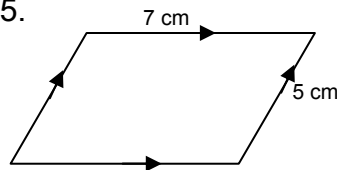
3.



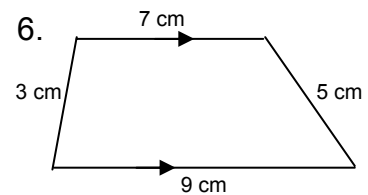
4.



5.



6.



Solutions:

$$\begin{aligned} 1. \quad P &= (3 + 5 + 6)\text{cm} \\ &= 14 \text{ cm} \end{aligned}$$

$$\begin{aligned} 2. \quad P &= [2(6) + 4] \text{ cm} \\ &= 12 + 4 \\ &= 16 \text{ cm} \end{aligned}$$

$$\begin{aligned} 3. \quad P &= 3(5 \text{ cm}) \\ &= 15 \text{ cm} \end{aligned}$$

$$\begin{aligned} 4. \quad P &= 4(6 \text{ cm}) \\ &= 24 \text{ cm} \end{aligned}$$

$$\begin{aligned} 5. \quad P &= [2(7\text{cm}) + 2(5\text{cm})] \\ &= 14\text{cm} + 10\text{cm} \\ &= 24\text{cm} \end{aligned}$$

$$\begin{aligned} 6. \quad P &= (3 + 7 + 5 + 9)\text{cm} \\ &= 24\text{cm} \end{aligned}$$

Example 2. Find the perimeter of the following regular polygons given the measure of a side (s).

1. triangle, $s = 10 \text{ cm}$

2. square, $s = 7\text{cm}$
3. pentagon, $s = 13.5\text{cm}$
4. hexagon, $s = 11.25\text{ cm}$
5. nonagon, $s = 9.3\text{ cm}$

Solutions:

$$\begin{aligned} 1. P &= 3s \\ &= 3(10\text{cm}) \\ &= 30\text{ cm} \end{aligned}$$

$$\begin{aligned} 2. P &= 4s \\ &= 4(7\text{cm}) \\ &= 28\text{cm} \end{aligned}$$

$$\begin{aligned} 3. P &= 5s \\ &= 5(13.5\text{cm}) \\ &= 67.5\text{ cm} \end{aligned}$$

$$\begin{aligned} 4. P &= 6s \\ &= 6(11.25\text{cm}) \\ &= 67.50\text{cm} \end{aligned}$$

$$\begin{aligned} 5. P &= 9s \\ &= 9(9.3\text{cm}) \\ &= 83.7\text{cm} \end{aligned}$$

Example 3 The width of a rectangular garden is 5m. If the length is 2m more than its width, how many meters of fencing materials are needed to enclose the whole garden?

How much will the cost of fencing material be if the owner pays P59 per meter?

Solution:

$$\begin{aligned} \text{Width (w)} &= 5\text{m} \\ \text{Length (l)} &= 2\text{m more than the width} \\ \text{Length (l)} &= 5\text{m} + 2\text{m} = 7\text{m} \end{aligned}$$

Get the perimeter:

$$\begin{aligned} P &= 2(l + w) \\ &= 2(7\text{m} + 5\text{m}) \\ &= 2(12\text{m}) = 24\text{m} \end{aligned}$$

$$\begin{aligned} \text{Cost of materials} &= 24\text{m (P59 /m)} \\ &= \text{P}1416.00 \end{aligned}$$

Example 4. The perimeter of a regular pentagon is 120 m. What is the length of each side?

Solution:

Pentagon has 5 sides, and since it is a regular pentagon, the sides are equal.

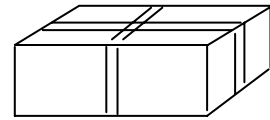
$$P = 5s$$

$$5s = 120 \text{ m}$$

$$s = \frac{120\text{m}}{5}$$

$$s = 24\text{m}.$$

Example 5. A saleslady is preparing to wrap a box whose dimensions are 30cm by 20cm by 7cm. If she is going to tie a ribbon around the box as in the figure, how long should the ribbon be if she will allow 25cm for the design at the top? How much will the ribbon cost if a meter is P8.25.



Solution: To find the length of the ribbon, find the two perimeters.

$$\begin{aligned} P_1 &= 2(30\text{cm}) + 2(7\text{cm}) \\ &= 60\text{cm} + 14\text{cm} \\ &= 74 \text{ cm} \end{aligned}$$

$$\begin{aligned} P_2 &= 2(20\text{cm}) + 2(7\text{cm}) \\ &= 40\text{cm} + 14\text{cm} \\ &= 54 \text{ cm} \end{aligned}$$

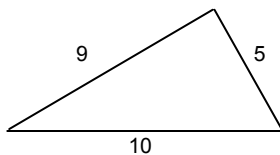
$$\begin{aligned} \text{Total length} &= P_1 + P_2 + 25\text{cm} \\ &= 74\text{cm} + 54\text{cm} + 25\text{cm} \\ &= 128\text{cm} = 1.28 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= 1.28 \text{ m}(P8.25) \\ &= P12.6225 \approx P12.65 \end{aligned}$$

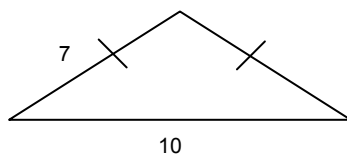
Try this out

A. Find the perimeter of the following using the given information in the figure.

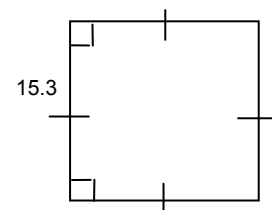
1.

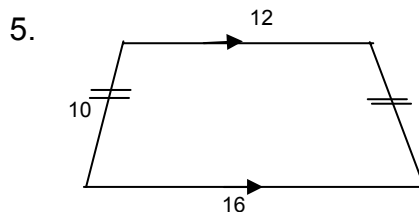
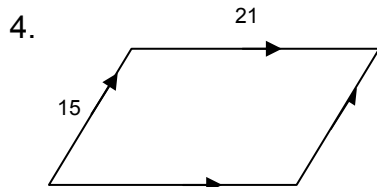


2.



3.





B. The sides of an isosceles triangle are given. Find the perimeter.

1. 8, 8, 13
2. 5, 4, 4
3. 7, 7, 5
4. 6, 7, 6
5. 1.2, 1.2, 1.3
6. 8.13, 8.14, 8.13
7. 12.75, 12.75, 10
8. 1, 2, 2

C. Given are the length and width of a rectangle. Find its perimeter.

1. 4.5, 8.3
2. 12.01, 19.22
3. 18.3, 21.5
4. 2.03, 5.43
5. 9.75, 12.25
6. $\sqrt{3}$, $5\sqrt{3}$
7. $4\sqrt{5}$, $8\sqrt{5}$
8. a , $3a$
9. $x + 1$, $5x + 4$
10. $x^2 + 1$, $x^2 + 25$

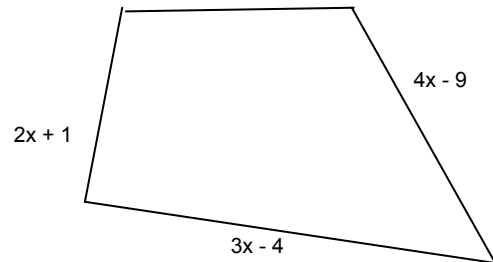
D. Find the perimeter of the following

1. A square with side of 25cm.
2. A parallelogram whose consecutive sides are 21cm and 27.5 cm respectively.
3. A regular hexagon whose side is $(2\sqrt{3} + 7)$ cm.
4. A rectangle 17m by 11m.
5. A rhombus with side of 18.23dm.

E. Solve the following problems.

1. The length of a rectangle is 4 less than three times its width. If the perimeter of the rectangle is 272, what are its dimensions?

2. The perimeter of a regular pentagon is 215 cm. Find the length of each side.
3. In an isosceles trapezoid, the length of one leg is 53dm and the median is 63dm. Find its perimeter.
4. James always jogs around his rectangular pool, 10m by 6m. If he jogs around it 5 times, what is the distance covered by James?
5. Find the perimeter of the given quadrilateral.



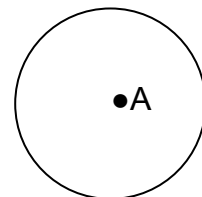
6. When the side of a square is increased by 2cm, its perimeter becomes 40 cm. What is the length of the original square?
7. When the side of a square is reduced by 7cm, the perimeter becomes 84. What is the perimeter of the original square?
8. The length of a rectangle is 64 cm. Its width is 13cm more than half its length. What is the perimeter of the rectangle?
9. The base of an isosceles triangle is 15 more than one-third of the length of the leg. If a leg measures 57 dm, find the perimeter of the triangle.
10. If the side of a square is increased by 25%, by how many percent will the perimeter increase?

Lesson 2

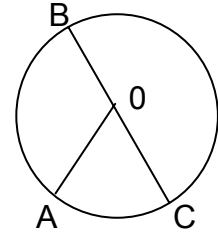
Circle and its Circumference

A circle is defined as the set of points equidistant from a fixed point called the center. Though the center is not a part of the circle, it is essential that every circle has a center. A circle on a plane can be represented geometrically or algebraically. In this part of the lesson, geometric circles will be the focus of our discussion.

In the given circle, A is the center of the circle, thus we can name the circle, circle A. There are other lines and segments associated with a circle like radius, chord, diameter, secant and tangent.

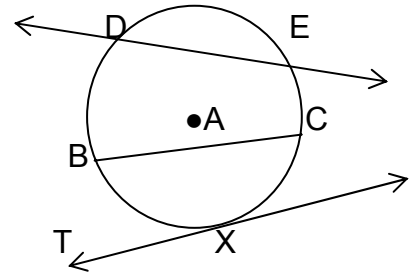


In circle O, \overline{OA} is a segment from the center to the circle. \overline{OA} therefore is a radius of the circle. Aside from \overline{OA} , other radii are \overline{OB} and \overline{OC} . \overline{BC} is a segment whose endpoints are points on the circle and it passes through the center of the circle. Therefore, \overline{BC} is called the diameter of the circle. If the endpoints of a segment are points on the circle, it is called chord of the circle.



By this definition, the diameter is also a chord of a circle. By inspection, it is very easy to recognize that the length of the diameter is twice that of the radius. Therefore, in the figure, $\overline{BC} = \overline{OB} + \overline{OC}$.

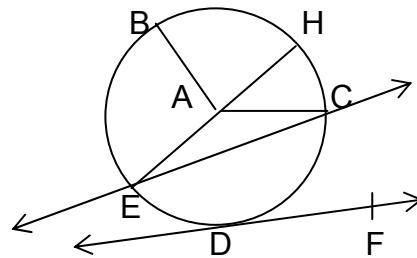
Other lines and segments associated with a circle are shown in the figure. They are \overline{BC} , \overleftrightarrow{DE} and line t . \overline{BC} has its endpoints on the circle. Thus it is an example of chord of the circle. It is not a diameter since it did not pass through the center of the circle. \overleftrightarrow{DE} intersects the circle at two points D and E. It is called a secant of the circle. Line t intersects the circle at only one point X. Line t is called tangent of the circle.



Example 1. Given circle A and points B, H, C, D and E on it.

Name:

1. 4 radii
2. 2 chords
3. a diameter
4. a secant
5. a tangent



Solutions:

1. $\overline{AB}, \overline{AC}, \overline{AH}, \overline{AE}$
2. $\overline{EH}, \overline{EC}$
3. \overline{EH}
4. \overleftrightarrow{EC}
5. \overleftrightarrow{DF}

Every polygon has its own perimeter. Likewise a circle has its own perimeter too. To distinguish it from those of polygons, we call the distance around the circle circumference. There is also a formula for finding the circumference of a circle. The letter "C" will be used to represent the circumference of the circle. The formula that we will use is

$C = 2\pi r$, where r is the radius of the circle and π is an irrational number whose value is approximated at 3.1416 or $\frac{22}{7}$.

An alternative formula for circumference can be used utilizing the diameter instead of the radius of the circle.

$C = \pi D$, where D is the length of the diameter of the circle.

Example 1. Find the circumference of a circle of radius 9cm.

Solution:

$$C = 2\pi r$$

$$C = 2\pi(9\text{cm})$$

$$C = 18\pi\text{cm}$$

If there is assigned value for π , say $\frac{22}{7}$, then

$$C = 2\left(\frac{22}{7}\right)(9\text{cm})$$

$$C = \frac{396}{7}\text{cm}$$

$$C \approx 56.57\text{cm}$$

If $\pi \approx 3.1416$, then

$$C = 2(3.1416)(9\text{cm})$$

$$C \approx 56.55\text{cm}$$

Example 2. The diameter of a circle is 12.6 dm. What is its circumference? Express the answer in terms of π .

Solution:

$$C = \pi D$$

$$C = \pi(12.6\text{dm})$$

$$C = 12.6\pi\text{dm}$$

Example 3. The circumference of a circle is 39.27cm. Find the radius of the circle. Use $\pi=3.1416$.

Solution:

$$C = 2\pi r$$

$$r = \frac{C}{2\pi}$$

$$r = \frac{39.27}{2(3.1416)}$$

$$r = 6.25\text{cm}$$

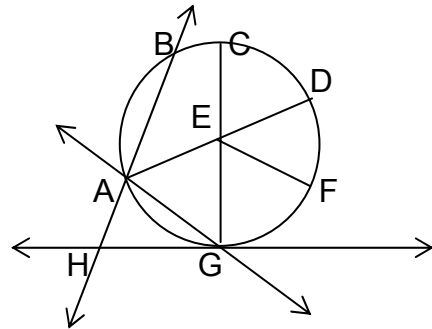
Try this out

A. Find the length of the diameter of a circle given its radius.

1. 24cm
2. 19 m
3. 3.96dm
4. .08 cm
5. $\frac{4}{7}$ dm
6. $\sqrt{18}$ m
7. $(x+2)$ cm
8. $\frac{x}{2}$ km
9. $\sqrt{m+n}$ m
10. $\frac{\sqrt{2a-b}}{4}$ dm

B. Use the figure and answer each of the following:

1. What is the center of the circle?
2. Name the circle.
3. Name 5 radii
4. Name 2 diameters
5. Name all the chords
6. Name a tangent of the circle
7. Name a secant



C. Find the circumference of the circle given the radius of the circle. Express answers in terms of π .

1. 19cm
2. 25dm
3. 11cm
4. 8cm
5. 13.75cm
6. 2.03cm
7. $\frac{5}{7}m$
8. $\sqrt{5}cm$
9. a dm
10. $(x + 3)cm$

D. Find the circumference of the circle if the diameter is given. Use $\pi = 3.1416$.

1. 24cm
2. 50dm
3. 35m
4. $\frac{3}{4}km$
5. 12.56m
6. 19.75cm
7. $\frac{1}{3}km$
8. 18.7cm
9. 25.76cm
10. 10.705cm
11. $\frac{2}{5}m$
12. 105.031cm
13. (a+b)dm
14. $(\sqrt{2} + 1)km$
15. $6\sqrt{5}$ dm
16. $3x^2$ cm
17. $(5x - 1)$ cm
18. $\sqrt{7x + 4}$ dm
19. 3.1416m
20. $\frac{3}{\sqrt{5} + 3}$ m

E. Given the circumference of the circle, find the radius and the diameter of the circle. Use the appropriate value of π .

1. 36π
2. 58π
3. 126π
4. 39π
5. 101π
6. 37.24π
7. 65.78π
8. 137.5π
9. 89.93π
10. 452.76π
11. 14.25
12. 549.78
13. 298.452
14. 314.16

15. 75.5
16. 78.54
17. $\sqrt{3}\pi$
18. 18456.9
19. $\sqrt{72} \pi$
20. $(10x - 4y) \pi$

F. Solve the following problems.

1. An artificial lake has a diameter of 34 m. If Teena jogs around it 6 times, how many meters will that be? (Use $\pi = 3.14$)
2. At one point in a race, Joseph was 15 m behind Allan and 18 m ahead of Brad. Brad was trailing Nick by 30m. Allan was ahead of Nick by how many meters?
3. What is the perimeter of a square inscribed in a circle of radius 10 cm?



Let's summarize

1. Perimeter of a polygon is the distance around the polygon.
2. The general formula for finding the perimeter of a polygon is

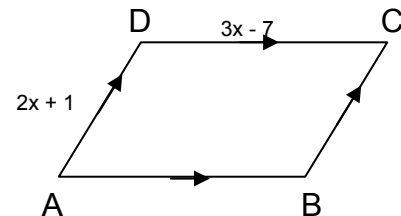
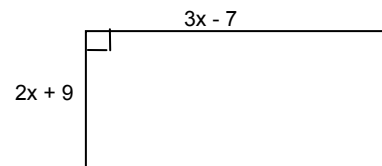
$$\text{Perimeter} = s_1 + s_2 + s_3 + \dots + s_{n-1} + s_n$$
 where s_n is the measure of the sides and n is the number of sides of the polygon.
3. For regular polygons, the perimeter is equal to the measure of a side multiplied by the number of sides.
4. A circle is the set of points equidistant from a fixed point called the center. The center of the circle defines the name of the circle.
5. Every circle must have a center.
6. Lines and segments associated with circle are the following:
 Radius – the segment joining the center and any point on the circle.
 Chord – segment joining any two points on the circle.
 Diameter – chord which passes through the center.
 Secant – a line intersecting a circle at two points.
 Tangent – a line intersecting a circle at only one point.
7. Circumference of a circle is the distance around a circle.

8. The formula for finding the circumference of a circle is $C = 2\pi r$, where r is the radius of the circle and π is an irrational number approximately equal to 3.1416 or $\frac{22}{7}$.



What have you learned

1. What is the perimeter of a regular heptagon with side of 23 cm?
2. The length of a leg of an isosceles trapezoid is 15 dm. The length of the median is 35dm. What is the perimeter of the trapezoid?
3. Find the perimeter of the given rectangle.
4. The circumference of a circle is 141π . Find the radius of the circle.
5. If the side of a square is increased by 4 cm, the perimeter becomes 136 cm. Find the length of the side of the original square.
6. The perimeter of a regular pentagon is 17.5 m. What is the length of each side?
7. What is the longest chord in a circle?
8. If the diameter of a circle is 25 cm, what is the length of the radius of the circle?
9. The base of an isosceles triangle is 27 dm. If the length of a leg is 11 more than one-third of the base, find the perimeter of the triangle.
10. ABCD is a parallelogram. Using the given in the figure, find x if the perimeter is 130 cm.





How much do you know

1. 52 cm
2. 40 cm
3. 46 dm
4. 46.25 cm
5. 76.38 cm
6. 10.5 cm
7. length = 43, width = 15
8. $8x + 4$
9. 43.96 cm
10. 11.5 cm

Lesson 1

A.

1. 24
2. 24
3. 61.2
4. 72
5. 48

B.

1. 29
2. 13
3. 19
4. 19
5. 3.7
6. 24.4
7. 35.5
8. 5

C.

1. 25.6
2. 62.46
3. 79.6
4. 14.92
5. 44
6. $12\sqrt{3}$
7. $24\sqrt{5}$
8. $8a$
9. $12x + 10$
10. $4x^2 + 52$

D.

1. 100 cm
2. 97 cm
3. $(12\sqrt{3} + 42)cm$
4. 56 m
5. 72.92 dm

E.

1. length = 101, width = 35
2. 43 cm
3. 232 dm
4. 160 m
5. $11x - 11$
6. 8 cm
7. 112 cm
8. 218 cm
9. 148 dm
10. 25%

Lesson 2

A.

1. 48 cm
2. 38 m
3. 7.92 dm
4. 0.16 cm
5. $\frac{8}{7}dm$
6. $6\sqrt{2}$ m
7. $(2x + 4)cm$
8. x cm
9. $2\sqrt{m+n}$ m
10. $\frac{\sqrt{2a-b}}{2}$ dm

B.

1. E
2. circle E
3. $\overline{EC}, \overline{ED}, \overline{EF}, \overline{EG}, \overline{EA}$
4. $\overline{AD}, \overline{CG}$
5. $\overline{AB}, \overline{AD}, \overline{CG}$
6. \overleftrightarrow{HG}
7. \overleftrightarrow{AB}

C.

1. 38π cm
2. 50π dm
3. 22π cm
4. 16π cm
5. 27.5π cm
6. 4.06π cm
7. $\frac{10}{7}\pi$ m
8. $2\sqrt{5}$ cm
9. $2a$ dm
10. $(2x + 6)$ cm

D.

1. 75.3984 cm
2. 157.08dm
3. 109.956 m
4. 2.3562 km
5. 39.458 m
6. 62.0466 cm
7. 1.0472 km
8. 58.74797 cm
9. 80.93 cm
10. 33.63 cm
11. 1.2566 m
12. 329.96 cm
13. $3.1416(a+b)$ dm
14. 7.58 km
15. 42.15 dm
16. $9.4248x^2$ cm
17. $(15.708x - 3.1416)$ cm
18. $3.1416(\sqrt{7x+4})$ dm
19. 9.8696 m
20. 1.8 m

E.

- | | | |
|----|---------------|-------------|
| 1. | $r = 18,$ | $d = 36$ |
| 2. | $r = 29,$ | $d = 58$ |
| 3. | $r = 63,$ | $d = 126$ |
| 4. | $r = 19.5$ | $d = 39$ |
| 5. | $r = 50.5,$ | $d = 101$ |
| 6. | $r = 18.62,$ | $d = 37.24$ |
| 7. | $r = 32.89,$ | $d = 65.78$ |
| 8. | $r = 68.75,$ | $d = 137$ |
| 9. | $r = 44.965,$ | $d = 89.93$ |

10. $r = 226.38,$ $d = 452.76$
11. $r = 2.268,$ $d = 4.536$
12. $r = 87.34,$ $d = 174.68$
13. $r = 47.5,$ $d = 95$
14. $r = 50,$ $d = 100$
15. $r = 12.02,$ $d = 24.04$
16. $r = 12.5,$ $d = 25$
17. $r = \frac{\sqrt{3}}{2},$ $d = \sqrt{3}$
18. $2937.5,$ $d = 5875$
19. $3\sqrt{2},$ $d = 6\sqrt{2}$
20. $5x - 2y,$ $d = 10x - 4y$

F.

1. 640.56 m
2. 3 meters
3. $40\sqrt{2}$ cm

What have you learned

1. 161 cm
2. 100 dm
3. $10x + 4$
4. 70.5
5. 30 cm
6. 3.5 m
7. diameter
8. 12.5 cm
9. 67 dm
10. 14.2 cm